

AD/RHIC-RD-5

Simulation of Transition and Transfer

(*Mini-Workshop on RHIC RF Systems*)

*July 11-15, 1988
Collider Center*

J. Wei
BNL

SIMULATION OF TRANSITION & TRANSFER

I. Transition Energy Crossing

- * $\hat{V} = 100 \text{ kV}$ crossing transition
- * Scheme of γ increase or δ_t jump

S. Y. Lee, A. G. Ruggiero
J. Claus

II. Transfer to High Frequency R.F. System

- * Top energy bunch rotation
- * Bunch rotation with $0.3 \text{ eV}\cdot\text{s}/\text{a bunch}$
- * Switch over near transition

J. M. Brennan, E. Rakai
S. Y. Lee ...

I. Transition Energy Crossing

- $\hat{V} = 1.2 \text{ MV}$?

momentum spread too large, nonlinear

- $\hat{V} = 100 \text{ kV}$ crossing :

much better,

space charge + nonlinear effect. $K_t(\frac{q}{p})$

⇒ phase space area blow up

- γ_t jump, or $\dot{\gamma}$ increase ?

Good, "clean" crossing

$$\begin{cases} \dot{\omega} = \frac{g e \tilde{V}}{2\pi} (\sin\phi - \sin\phi_s) + \Delta_{s.c.} + \Delta_z \\ \dot{\phi} = \frac{h \Omega_0}{P_0 R_0} \cdot \omega \cdot \eta(\omega) \end{cases} \quad (\omega = \frac{\Delta E}{R_0})$$

* Kinematic mismatching

$$\eta\left(\frac{\alpha p}{p}\right) = \eta_0 + \eta_1 \cdot \frac{\alpha p}{p} + \dots$$

* Low freq. impedance

space charge, inductive, capacitive

⇒ bunch mismatching

* High freq. impedance

resistive, inductive, capacitive (s.c.)

⇒ microwave instability

$$\underline{\eta = \eta(\frac{\Delta P}{P_0})}$$

$$\eta R = \beta c, \quad \frac{\Delta R}{R_0} = \frac{\beta}{\beta_0} \frac{R_0}{R} - 1$$

$$\frac{R}{R_0} = 1 + \alpha_1 \frac{\Delta P}{P_0} + \alpha_2 \left(\frac{\Delta P}{P_0} \right)^2 + \dots$$

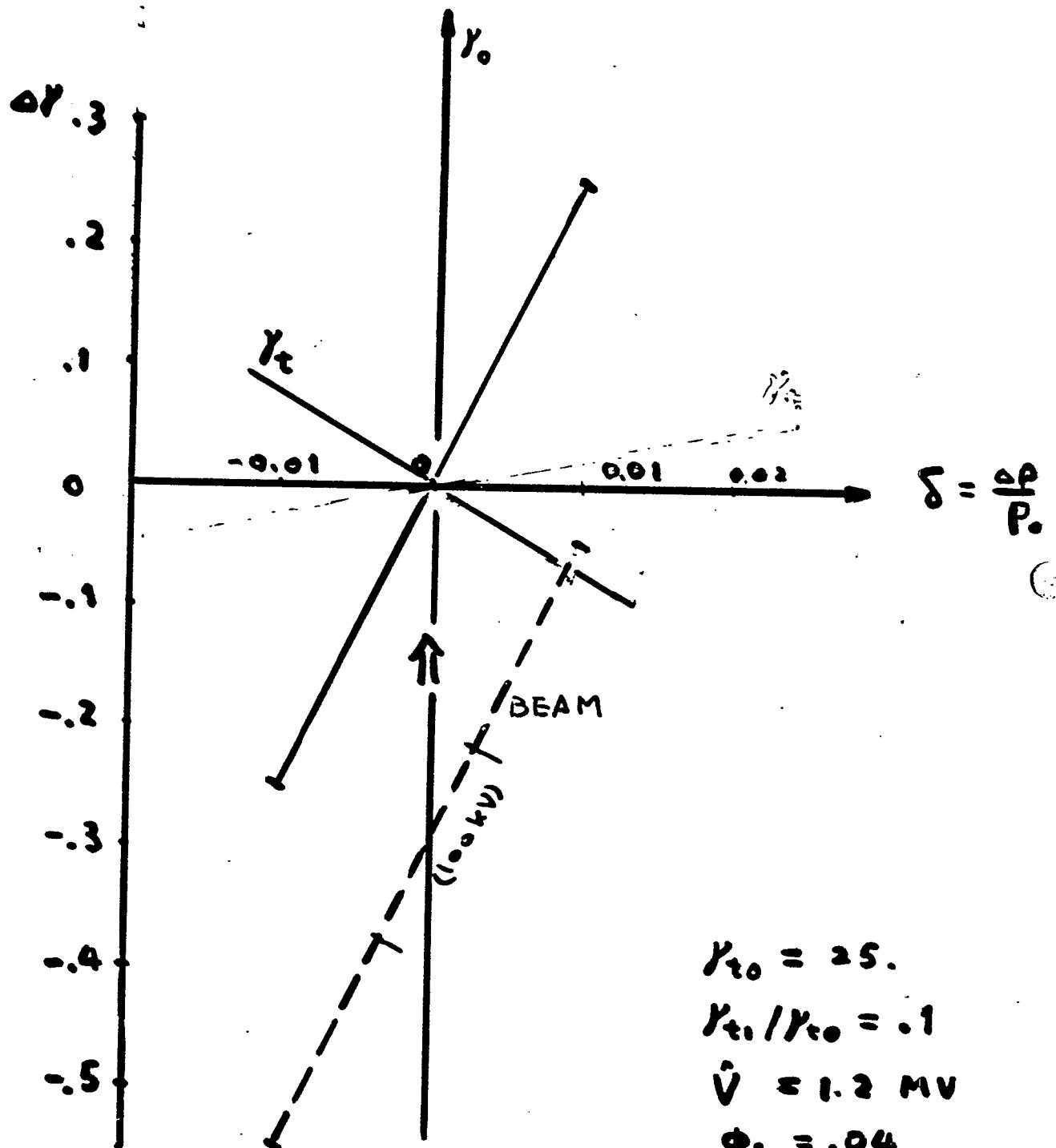
$$\frac{\beta}{\beta_0} = 1 + \frac{1}{\gamma_0^2} \cdot \frac{\Delta P}{P_0} - \frac{3}{2} \frac{\beta_0^2}{\gamma_0^2} \cdot \left(\frac{\Delta P}{P_0} \right)^2 + \dots$$

$$\underline{\eta = \frac{\Delta R}{R} / \frac{\Delta P}{P_0} \equiv \eta_0 + \left[\frac{3}{2} \frac{\beta_0^2}{\gamma_0^2} - \frac{\gamma_0}{\gamma_{+0}} \right] \cdot \frac{\Delta P}{P_0}}$$

$$\delta = \frac{\Delta \gamma}{\gamma} = \left(1 + \frac{1}{\beta_0^2 \gamma_0^2} \right) \cdot \frac{\Delta P}{P_0}$$

$$\left(\frac{\gamma}{\gamma_0} - \frac{\gamma_0}{\gamma_{+0}} \right)$$

Mismatch at transition (RHIC)



$$\gamma_{t_0} = 25.$$

$$\gamma_{t_1}/\gamma_{t_0} = .1$$

$$\dot{V} = 1.2 \text{ MV}$$

$$\Phi_s = .04$$

$$\Delta Y = 2.07 \cdot 10^5 \text{ turn}$$

$$\dot{\gamma} = 1.62/\text{sec.}$$

$$\tau_0 = 12.79 \mu\text{s}$$

Estimate of δ growth.

$$\ddot{\delta} = \frac{q e \dot{V}}{2\pi p_0 R_0} \cos \varphi_0 k \Omega_0 \left(\frac{2\dot{\gamma} t}{\gamma_1^{0.3}} + \eta, \delta \right) \delta$$

$$\int_{t_c}^0 \Omega_s(t) dt = \left(\frac{k \Omega q e \dot{V} \cos \varphi_0}{2\pi p_0 R_0} \right)^{1/2} \int_{t_c}^0 \left(\frac{2\dot{\gamma} t}{\gamma_1^{0.3}} + \eta, \delta \right)^{1/2} dt$$

$$\approx \frac{\Omega}{3\dot{\gamma}} \left(\frac{h q e \dot{V} \cos \varphi_0}{2\pi p_0 \rho c} \right)^{1/2} \left(\frac{3}{2} - \lambda \right)^{3/2} \hat{\delta}^{3/2}$$

AGS $295 \hat{\delta}^{3/2}$	RHIC $5200 \hat{\delta}^{3/2}$
Growth factor: $e \approx 1.1$	$e \approx 6.3$

$$(\hat{k} \hat{V} \cos \varphi_0)^{1/2} \hat{\delta}^{3/2}$$

$$\delta \propto \delta_0 e$$

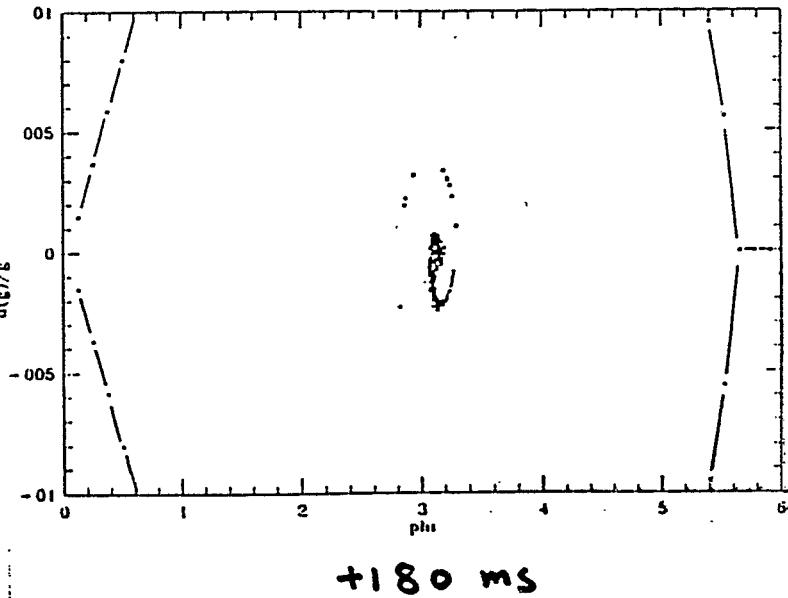
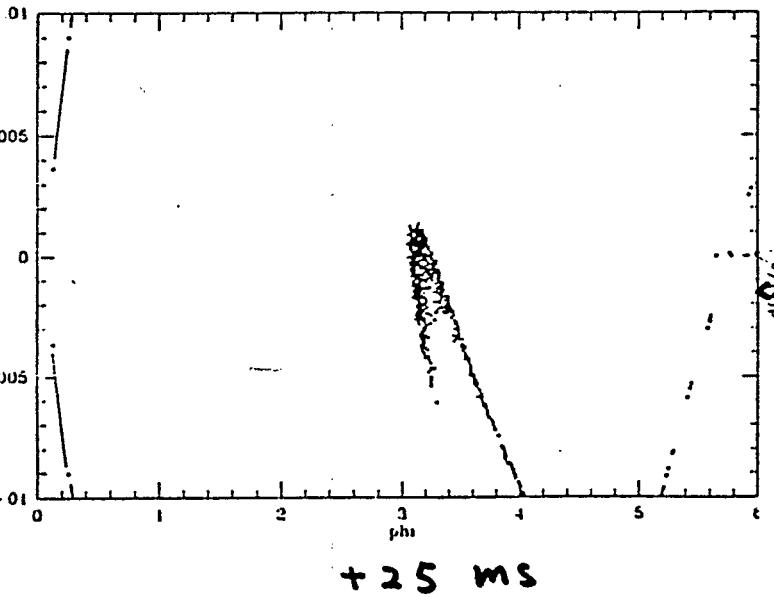
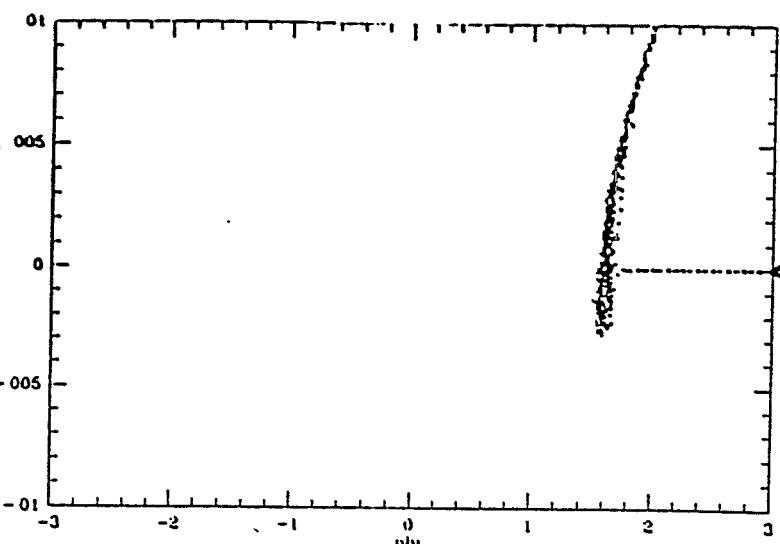
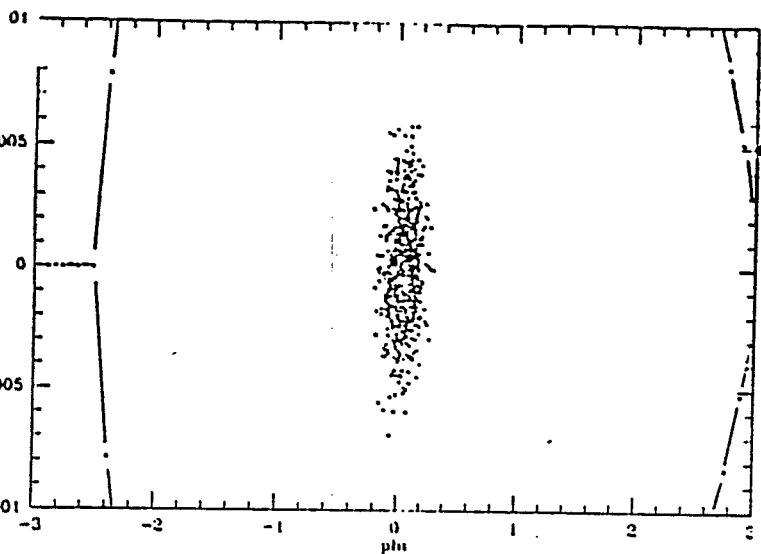
Relevant time scales:

- τ_0 : revolution period
- $\tau_{\text{syn.}}$: synchrotron osc. period
- T_c : characteristic non-adiabatic time $\frac{2}{\pi^2} \cdot \left| \frac{dR_s}{dt} \right| > 1$
$$T_c = \left[\frac{A m_e c^2}{R_0^2 h} \cdot \frac{\gamma_t^4}{2 \dot{\gamma}} \cdot \frac{2\pi}{zeV |\cos\phi_s|} \right]^{1/3}$$
- $T_{n.l.}$: non-linear time
$$T_{n.l.} = \frac{\Delta P_{n.l.}}{\dot{\gamma}} = \frac{3 \beta_s^2 - \alpha_1}{4 \dot{\gamma}} \cdot \left(\frac{\Delta P}{P} \right)_{z=50}$$
$$\eta(\delta) = \eta_0 + \eta_1 \delta + \dots$$
time when $|\eta_1 \delta| > 1$ or of different sign
- $T_{m.w.}$: microwave instability growth time

$\hat{V} = 1.2 \text{ MV}$
 $\sin\phi_s = 0.04$

γ_+ CROSSING

Au^{+79}



$\eta(\frac{\alpha p}{p})$, mismatching $\Rightarrow 70\%$ Loss

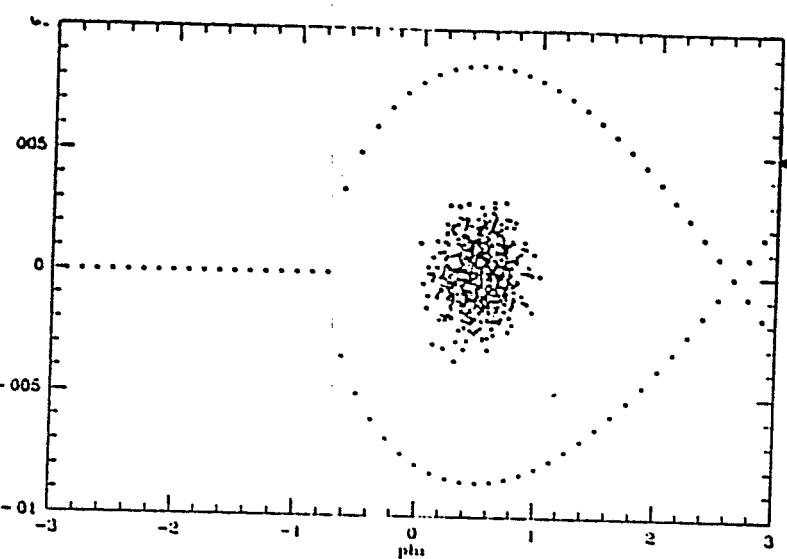
$\tau_0 \ll T_c < T_{n.l.} < \tau_{\text{syn.}}$

$V = 100 \text{ kV}$

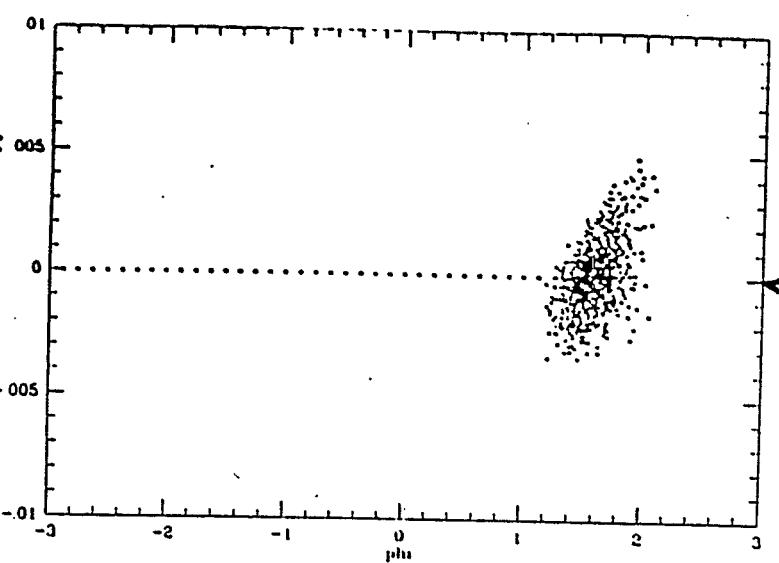
$\sin\phi_s = 0.48$

γ_t CROSSING

A_u^{+79}

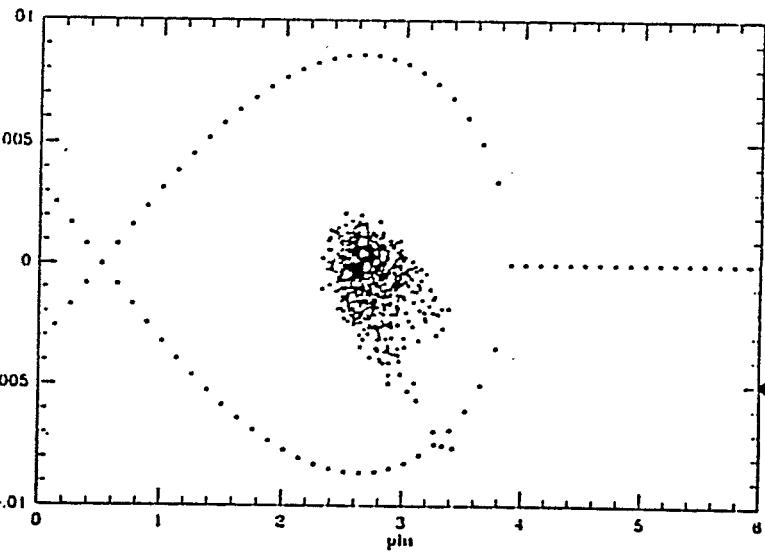


-80 ms

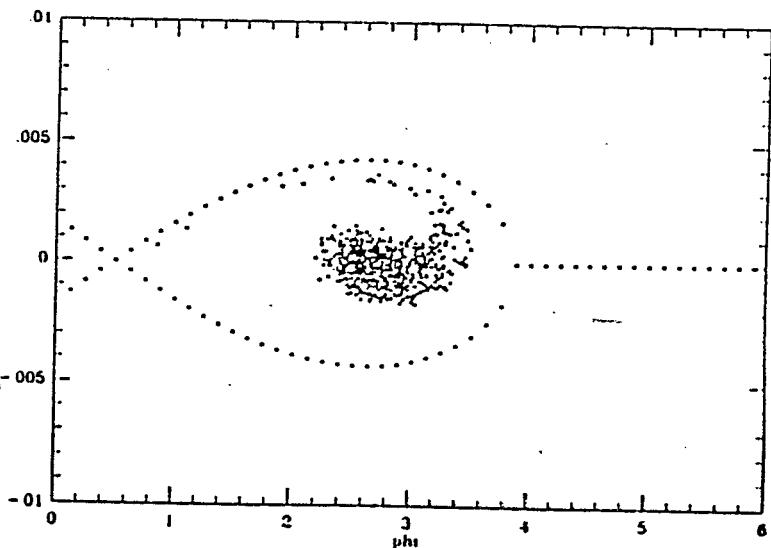


0 ms

$0.3 \text{ ev.s/gmu}, \text{ w/o s.c.}$



$+80 \text{ ms}$



$+230 \text{ ms}$

$\eta(\frac{\Delta p}{p})$ mismatching \Rightarrow 0.9% Loss

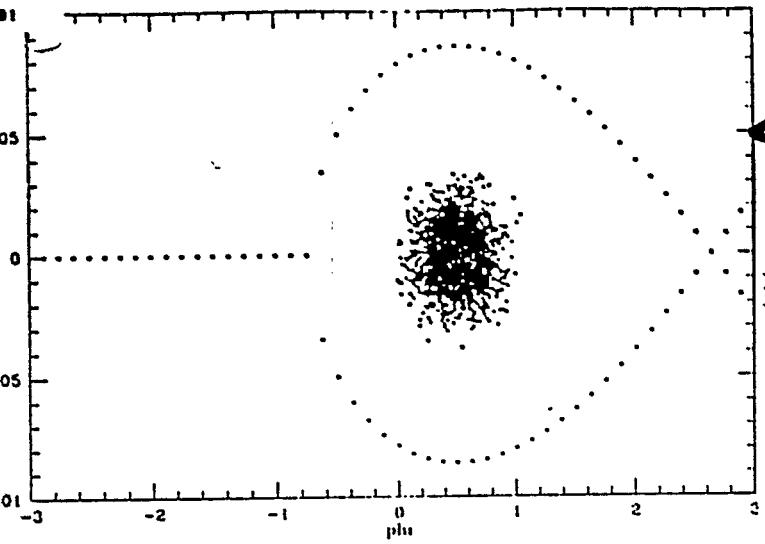
$\tau_0 \ll T_{n.l.} < T_c < \tau_{syn.}$

$V = 100 \text{ kV}$

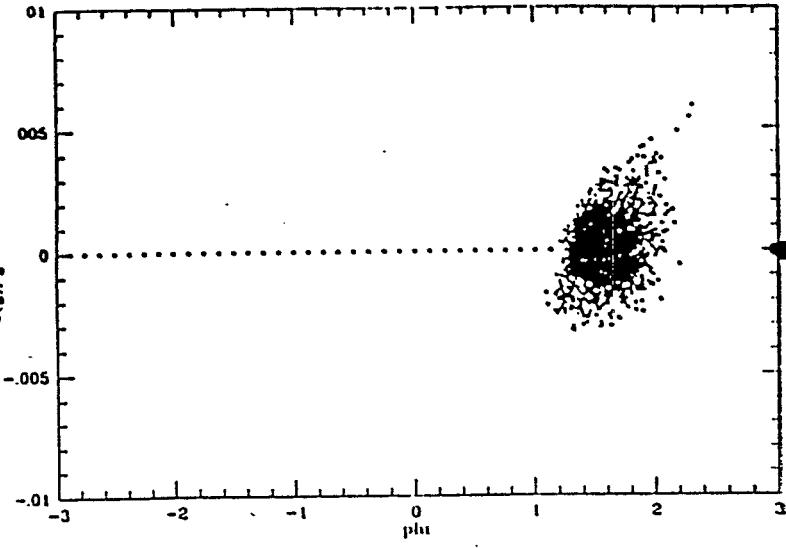
$\sin \phi_s = 0.48$

γ_t CROSING, space charge

Au



-80 ms

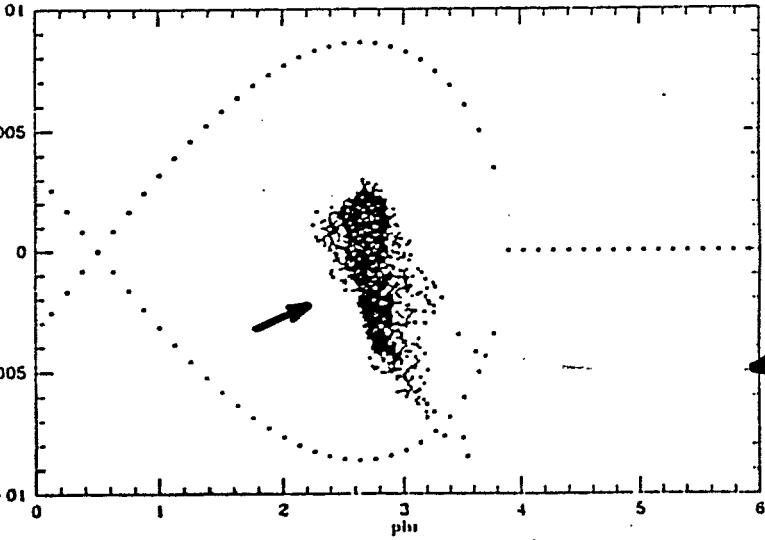


0 ms

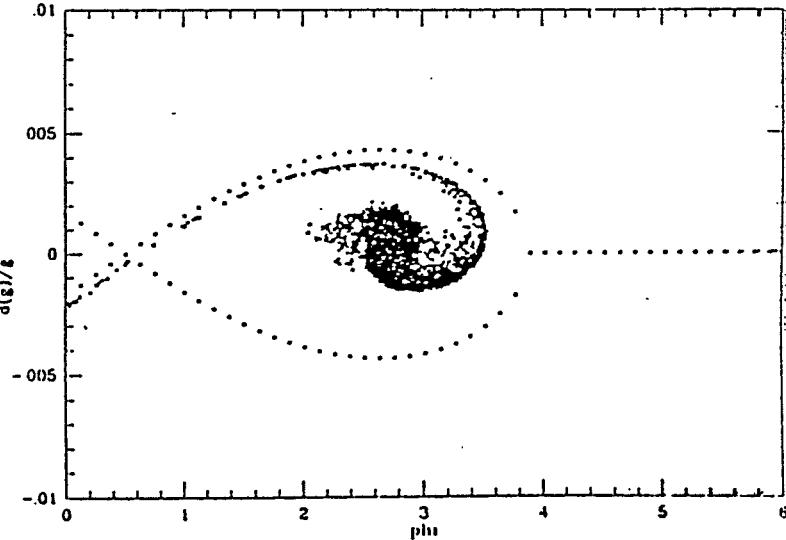
$0.3 \text{ eV}\cdot\text{s}/\text{amu}$, with s.c.

$\frac{Z}{n} = 1.2 \Omega$, $1.1 \times 10^9/\text{bunch}$

$l_{\text{bin}} = 2b$, \Rightarrow microwave freq.



$+80 \text{ ms}$



$+230 \text{ ms}$

mismatching + spacecharge \Rightarrow 2.1% Loss

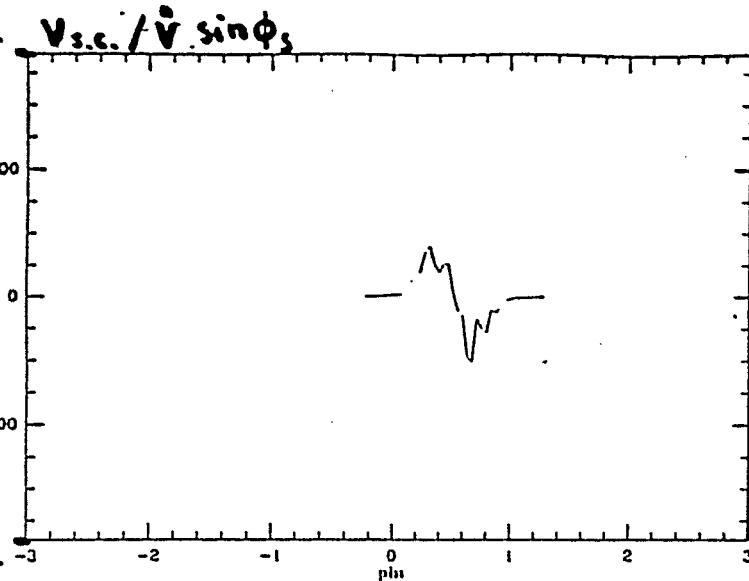
$\tau_0 \ll T_{\text{n.l.}} < T_c < \tau_{\text{syn.}}$

$\sim 1 \text{ ev}\cdot\text{sec}/\text{amu}$

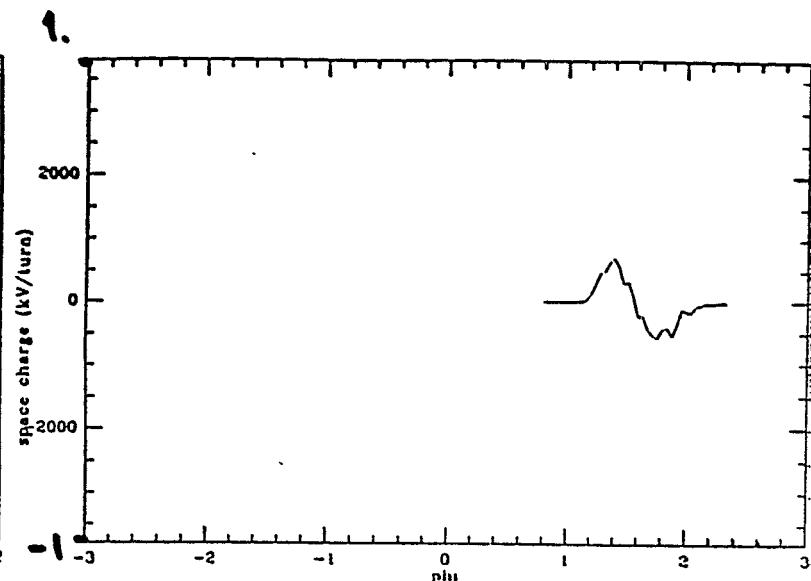
$$\hat{V} = 100 \text{ kV}$$

$$\sin\phi_s = 0.48$$

Space charge voltage

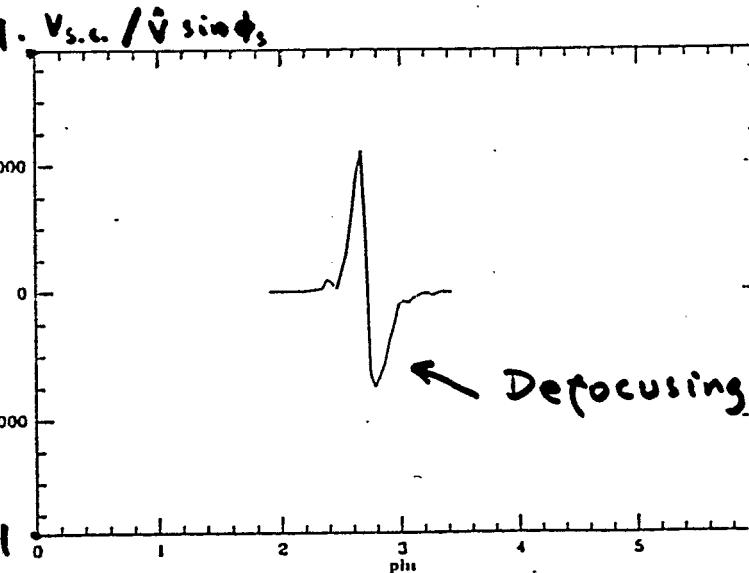


-80 ms

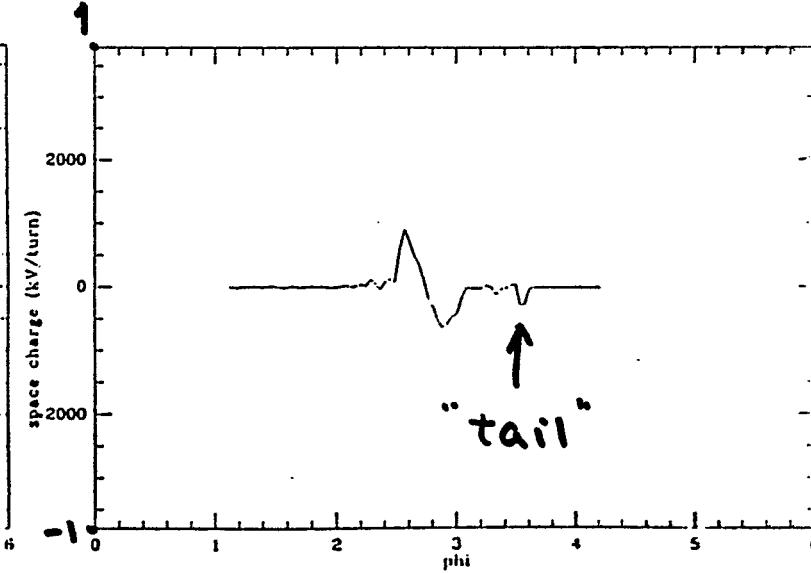


0 ms

$$\frac{Z}{n} = 1.2 \Omega, \text{ Capacitive}$$



+ 80 ms

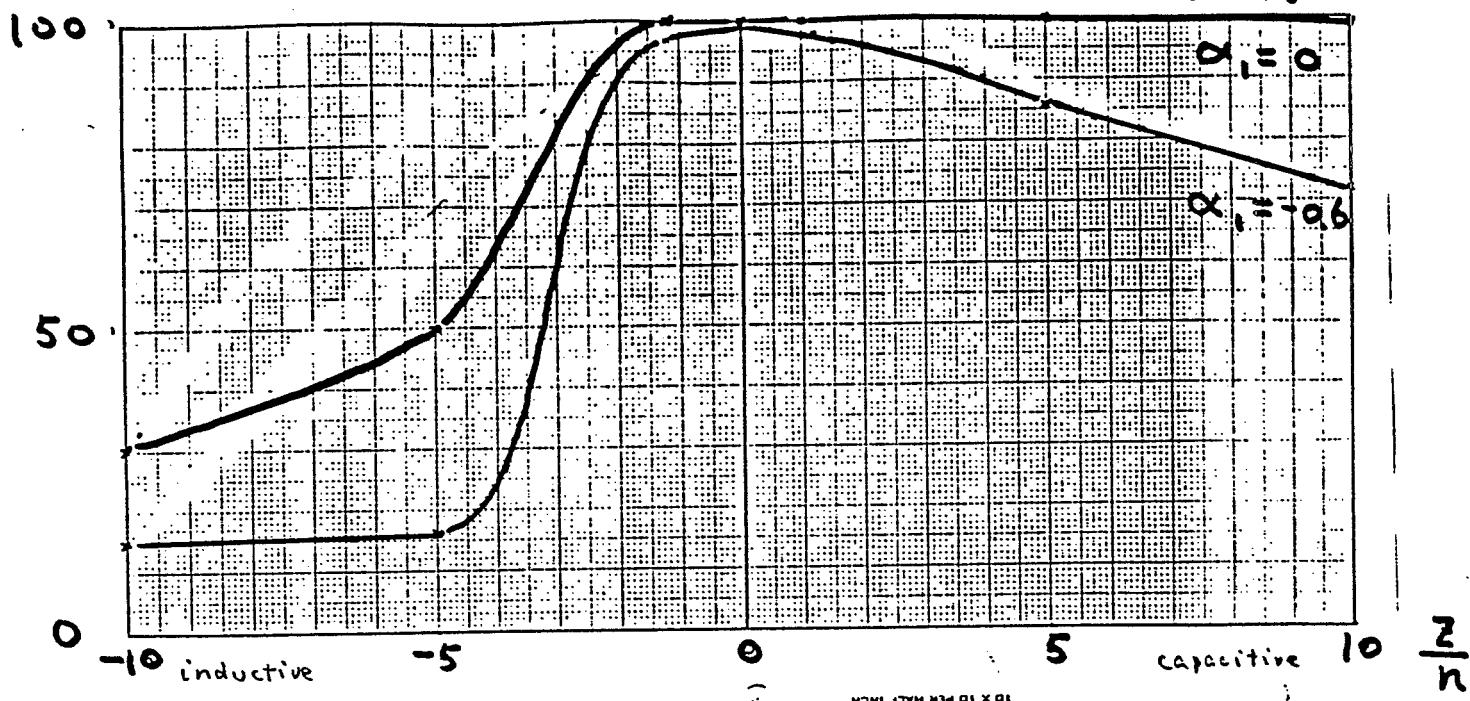


+ 230 ms

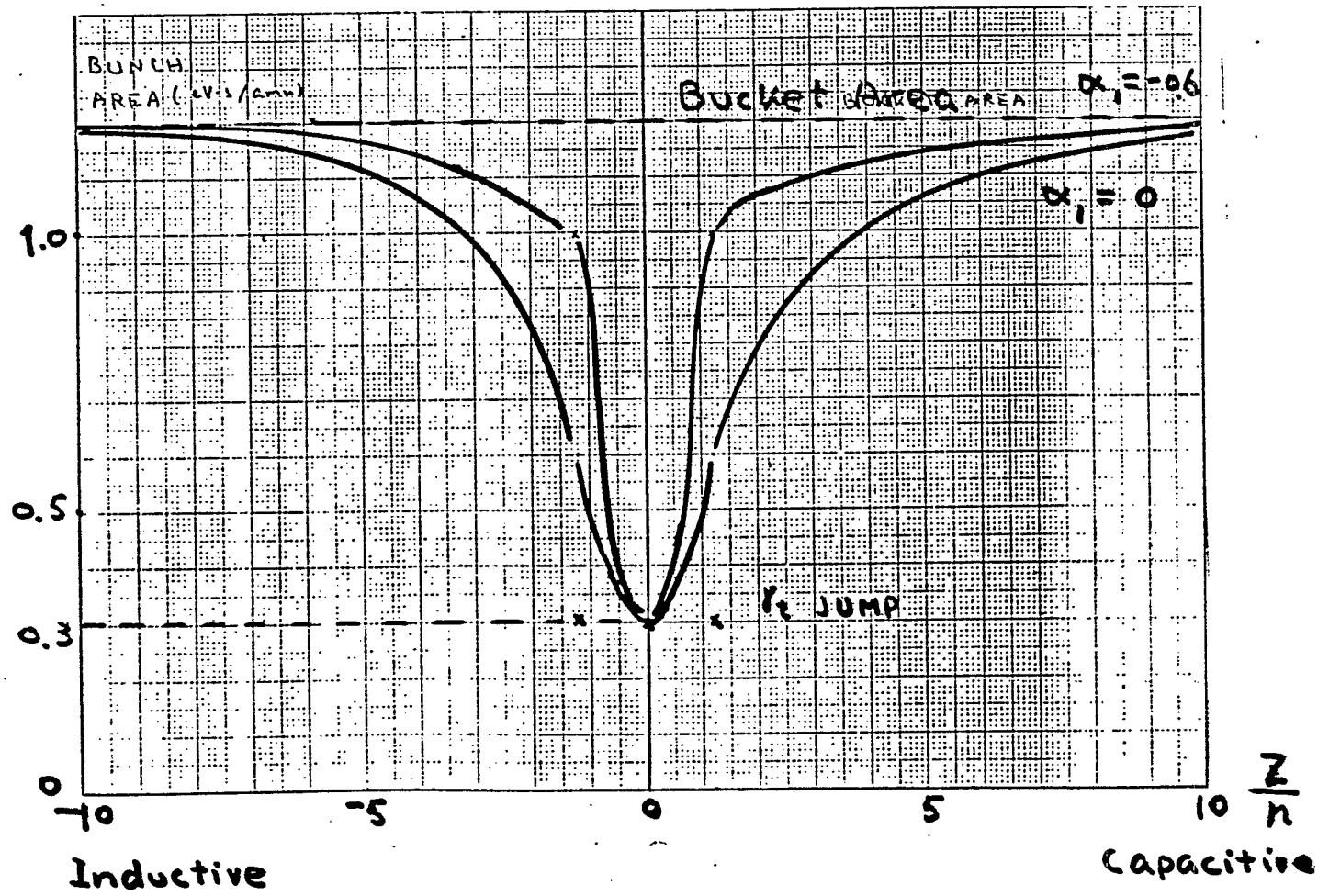
Defocusing space charge force

Crossing Efficiency (%)

Au^{79} . RHIC
 $\hat{V}_t = 100$ kV
 $\sin\phi_t = 0.48$



Bunch Area After Transition



S.Y.Lee & J.M.Wang

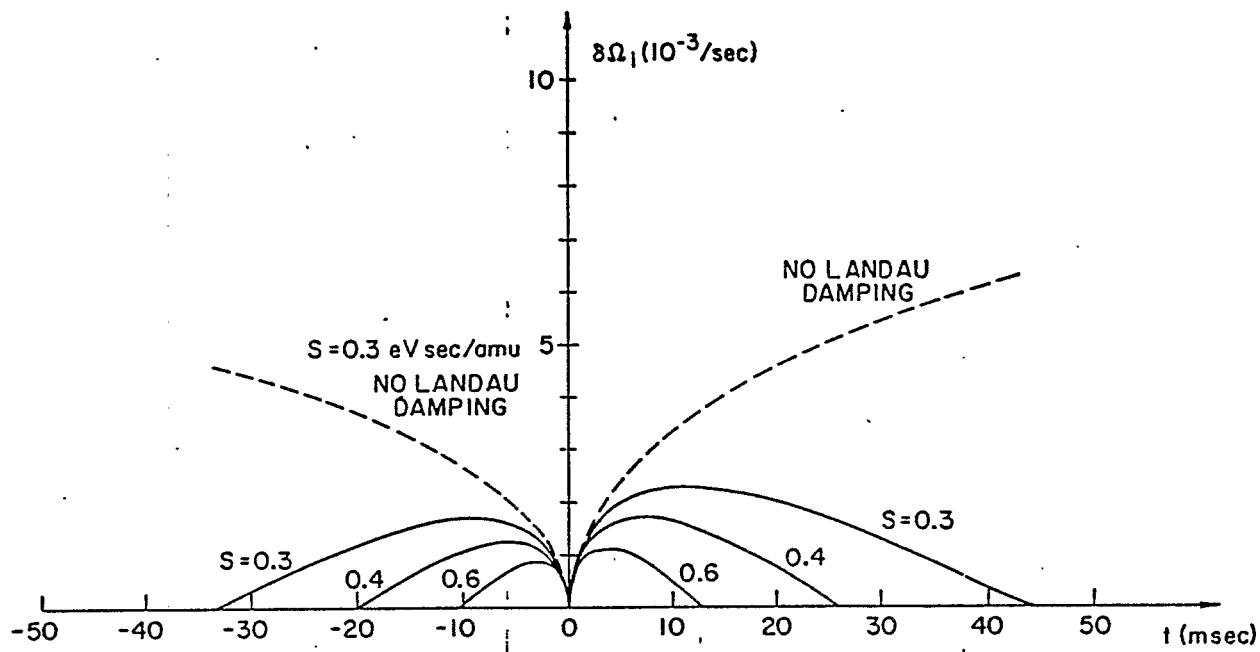


Fig. IV-19. The imaginary part of the microwave frequency, $\delta\Omega_i$, representing the growth rate of the instability, is plotted as function of time during crossing of the transition energy. The curves are calculated with the initial phase space as parameter. The dashed curve represents the growth rate without Landau damping.

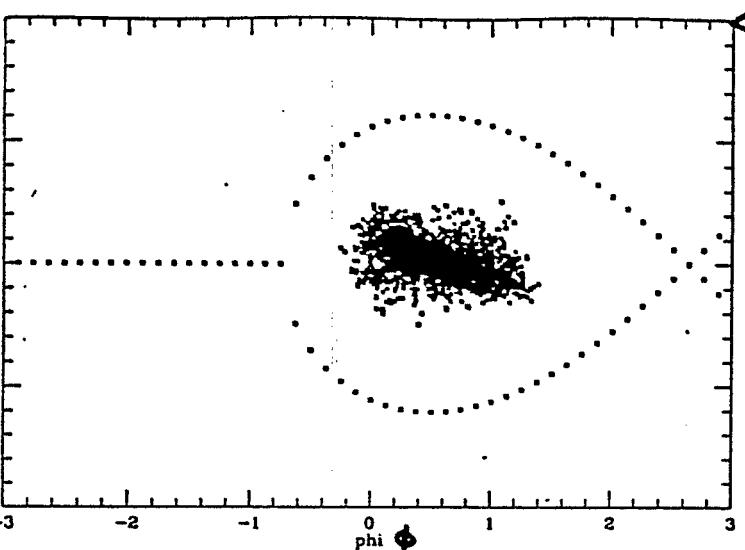
* Analytical solution based on
perturbation theory

$V = 100 \text{ kV}$
 $\sin\phi_s = 0.48$

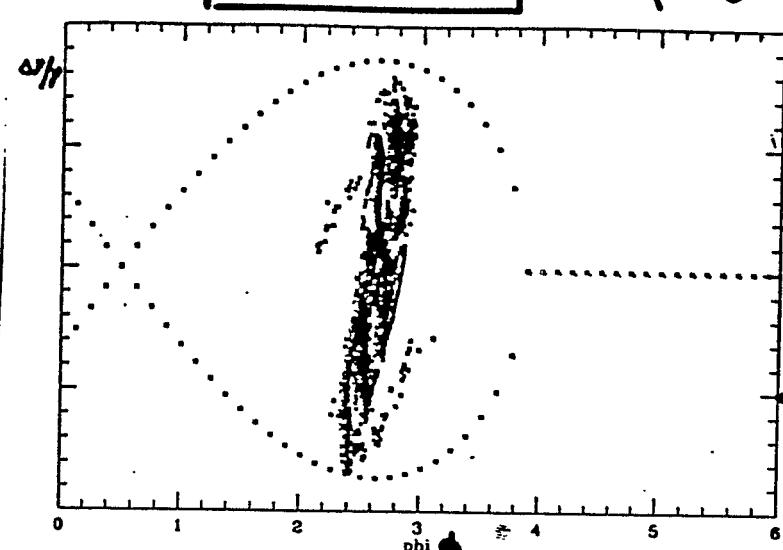
γ crossing

$Z/n = 10 \Omega$
capacitive

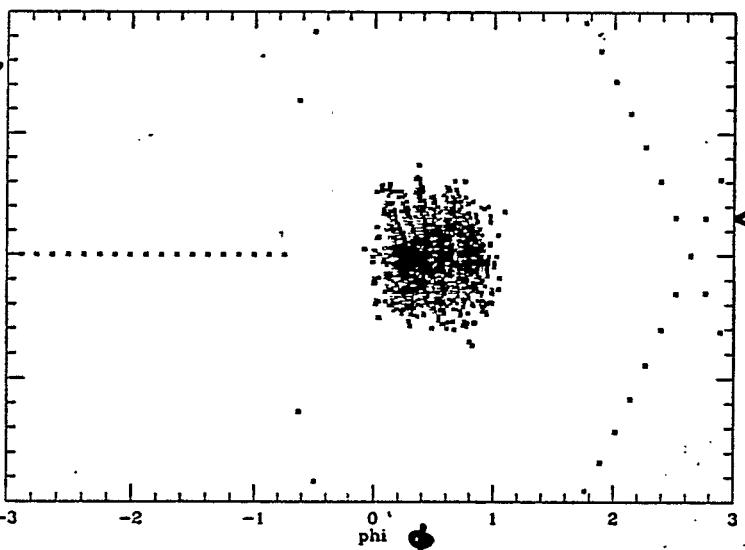
Au^{+79}
 $\alpha_s = 0$



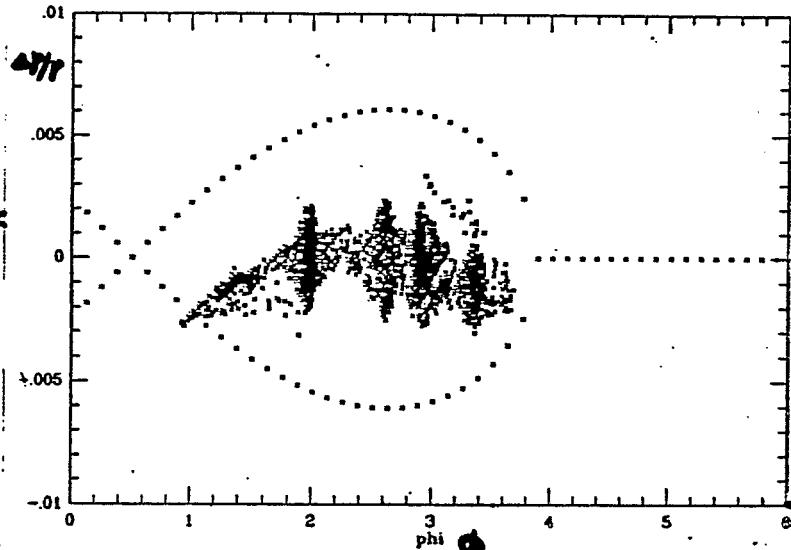
-160 ms



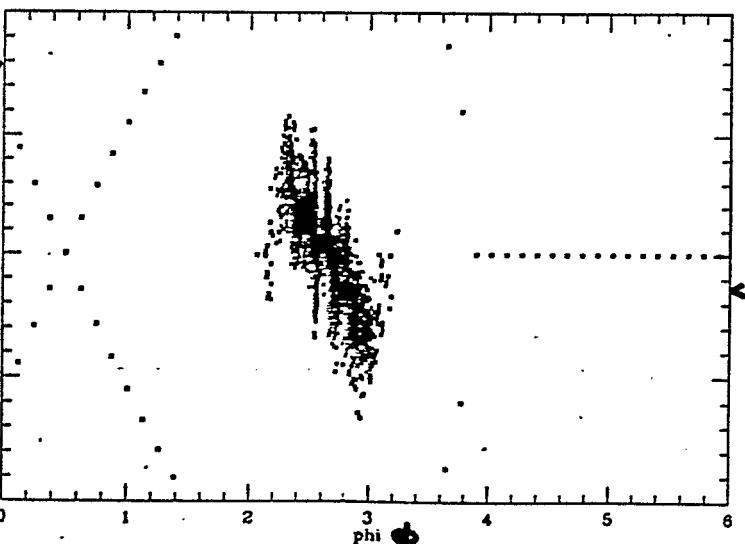
$+80 \text{ ms}$



-25 ms

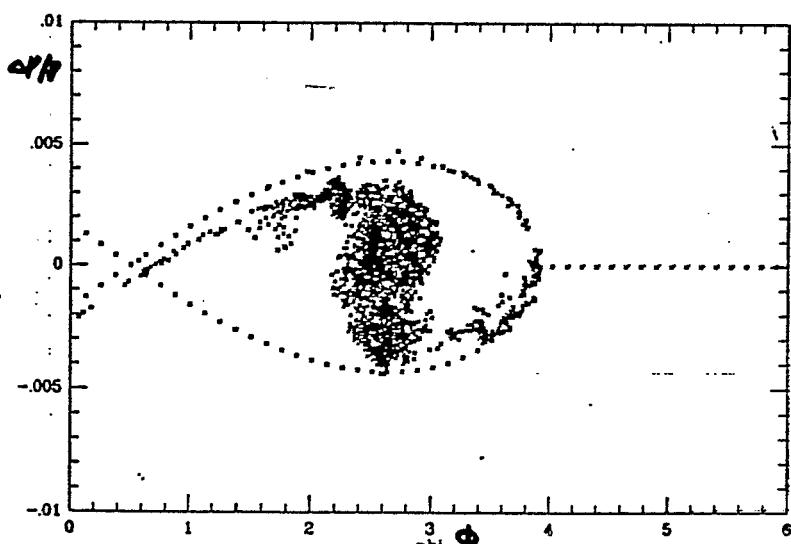


$+160 \text{ ms}$



$+2.5 \text{ ms}$

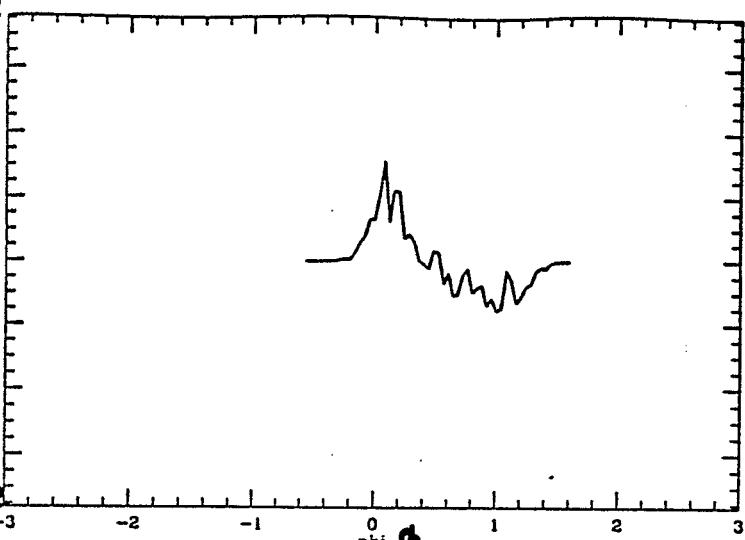
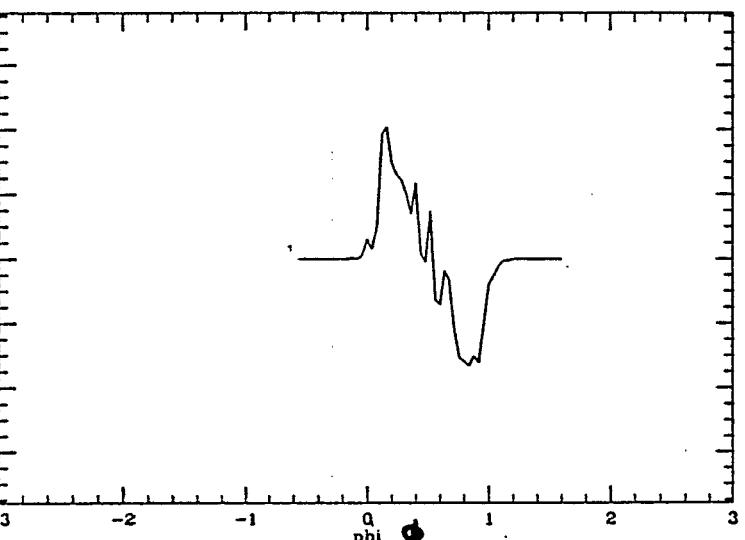
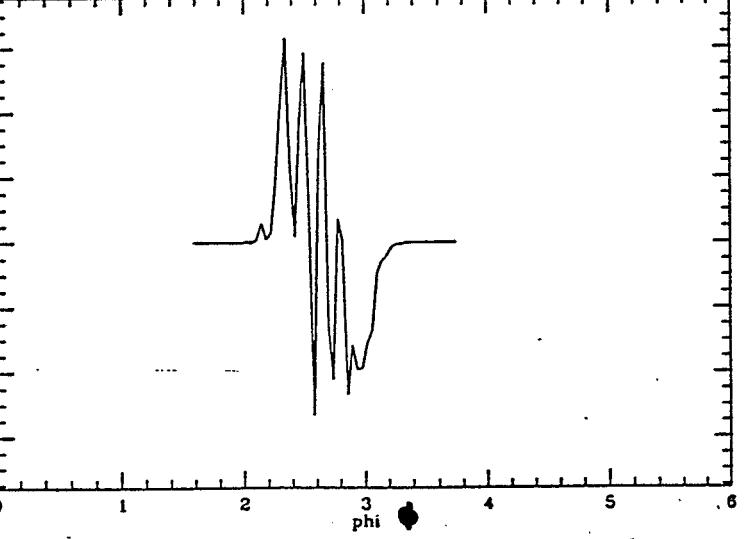
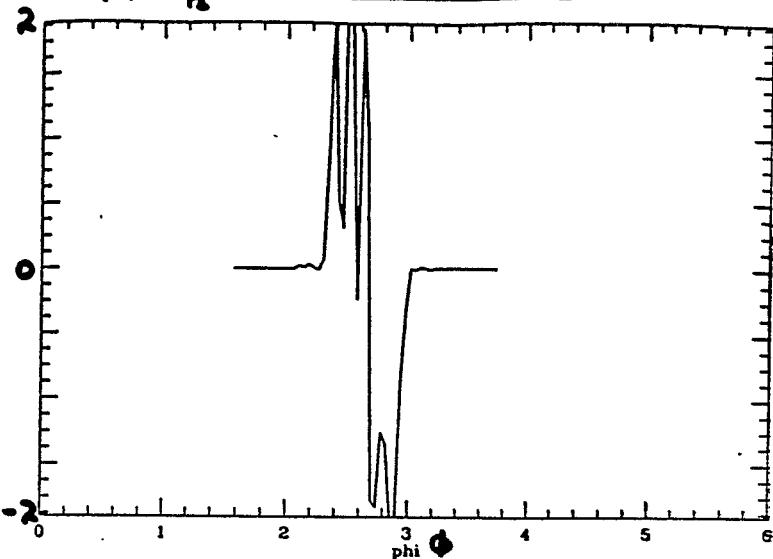
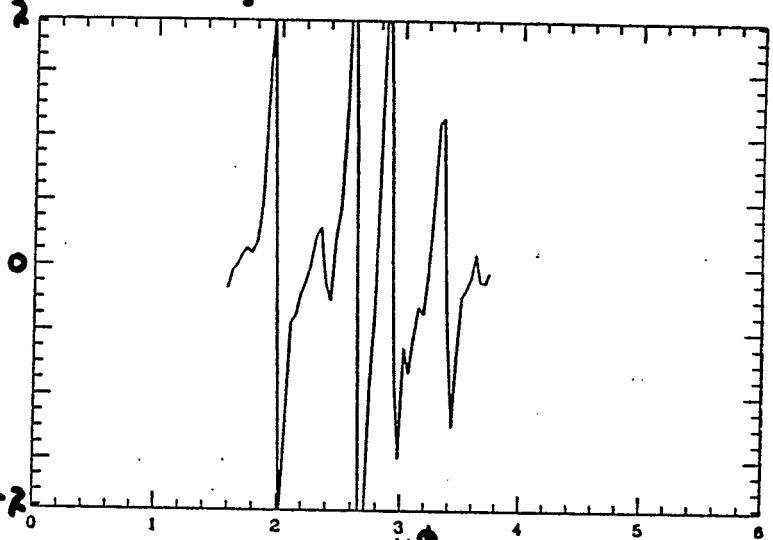
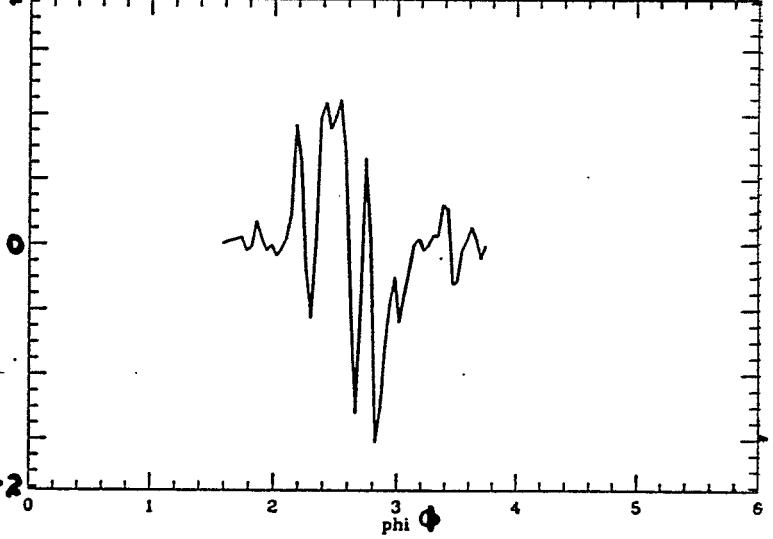
Efficiency 99%



$+320 \text{ ms}$

$Z/n = 10 \Omega$

capacitive

 Au^{+79} $\alpha_s = 0$ $V_{t.c.}/V \cdot \sin\phi_s$  -160 ms $V_{t.c.}/V \cdot \sin\phi_s$  -25 ms $V_{t.c.}/V \cdot \sin\phi_s$  $+25$ ms $V_{t.c.}/V \cdot \sin\phi_s$  $+80$ ms $V_{t.c.}/V \cdot \sin\phi_s$  $+160$ ms $V_{t.c.}/V \cdot \sin\phi_s$  $+320$ ms

To preserve the original $0.3 \text{ eV}\cdot\text{s}/\text{amu}$
& to minimize loss

- * γ_t jump, or
- * increase γ near transition
(without changing B), or
- * make total $\frac{Z}{n} \sim 0$

• Resistive wall impedance
very very small effect

$$10^2 * \left| \frac{Z}{n} \right|_{\text{stainless steel}} \Rightarrow \begin{array}{l} \text{microwave} \\ \text{instability} \end{array}$$

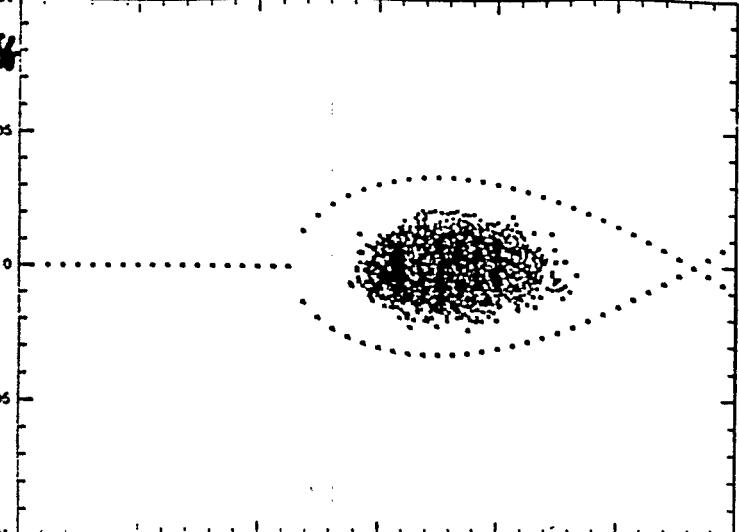
$$@ \frac{Z}{n_c} \sim 5 \Omega, \quad n_c = \frac{R_0}{b}$$

$V = 100 \text{ kV}$
 $\sin \phi_s = 0.48$

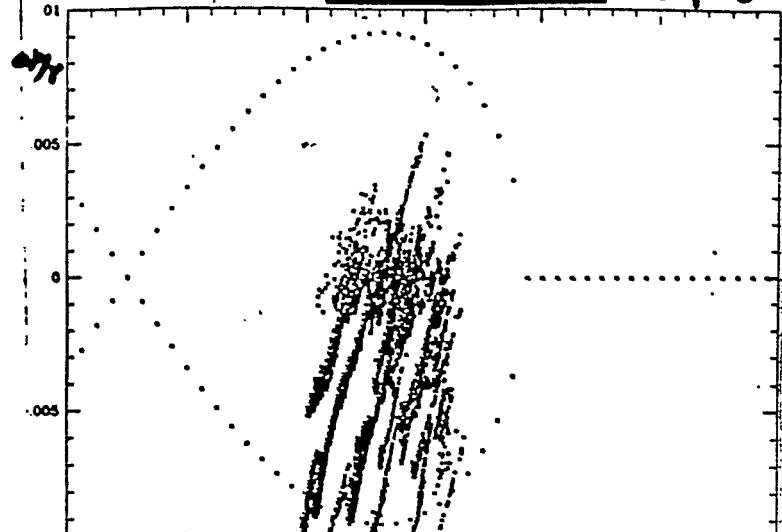
γ_+ crossing

$R = 10^3 \Omega$
resistive

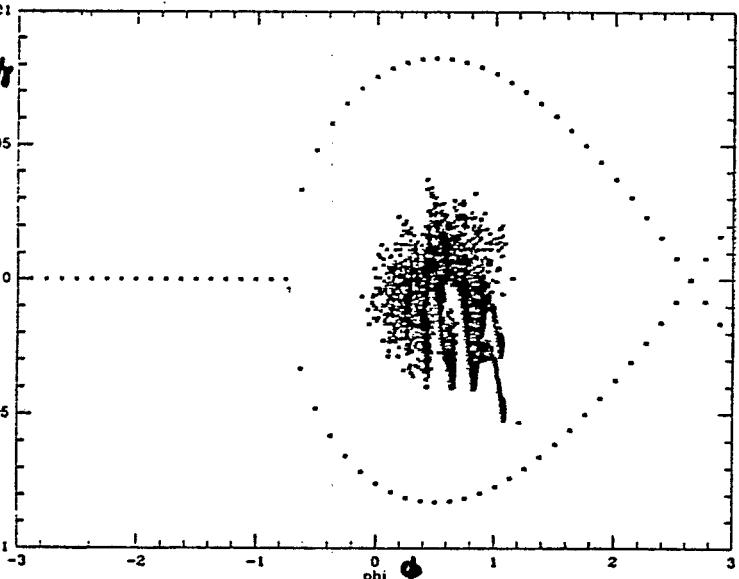
A_{eff}
 $\alpha_s = 0$



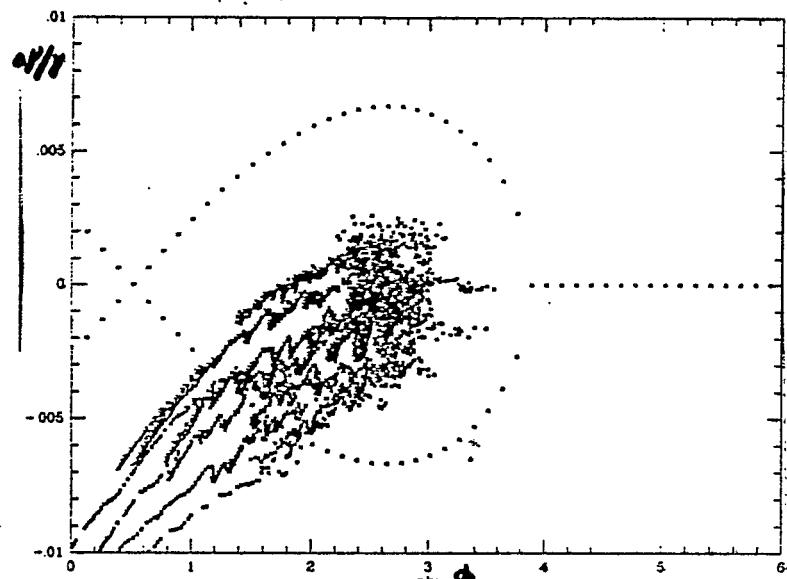
-520 ms



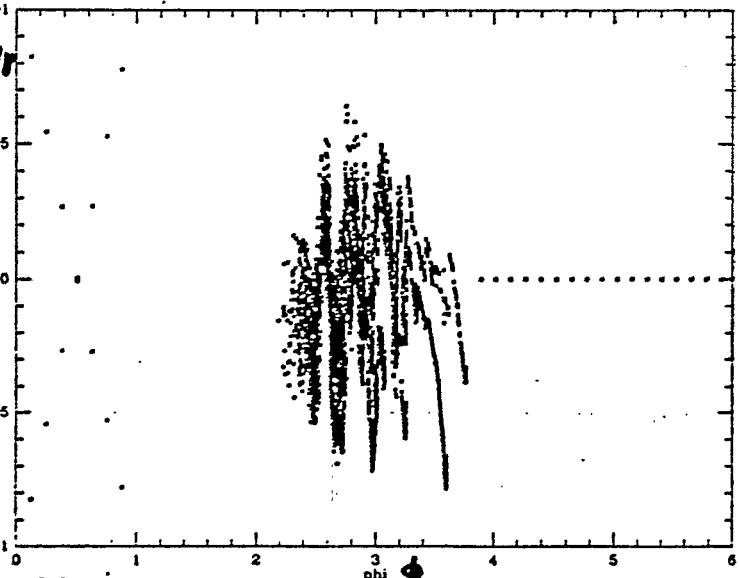
$+100 \text{ ms}$



-85 ms

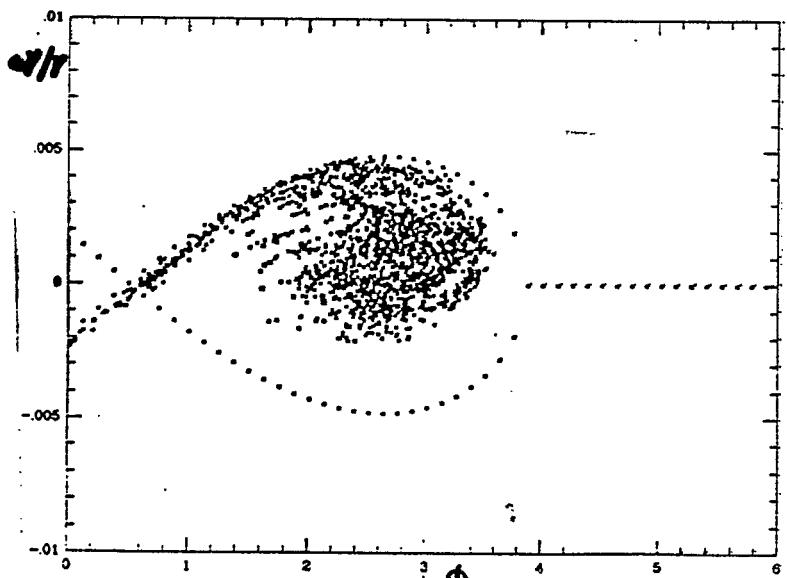


$+165 \text{ ms}$



$+40 \text{ ms}$

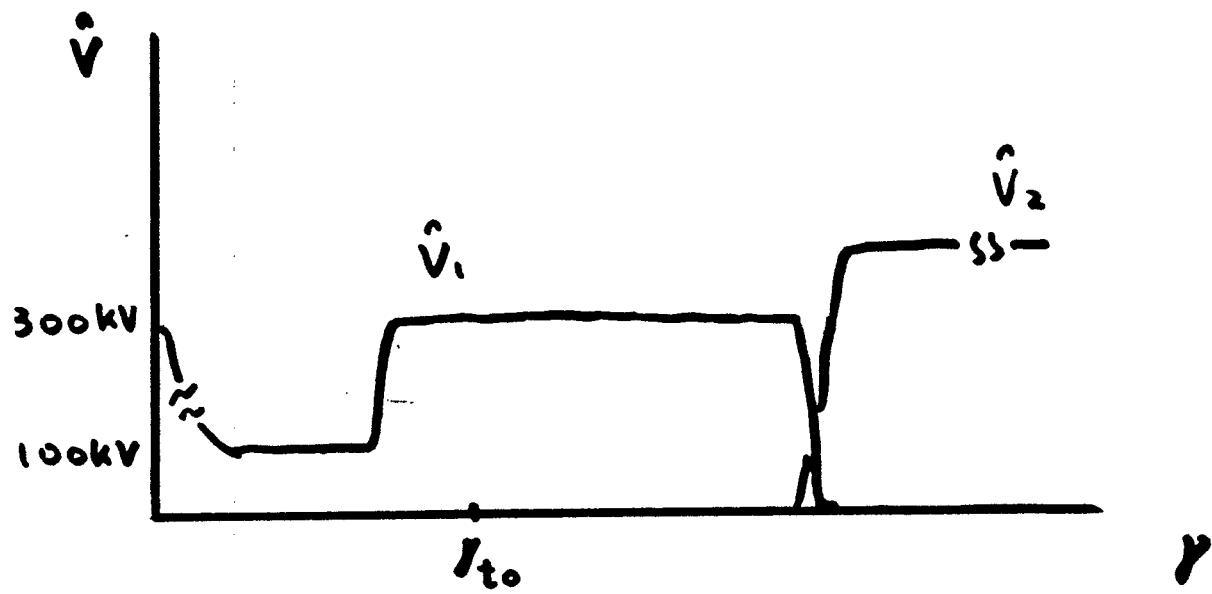
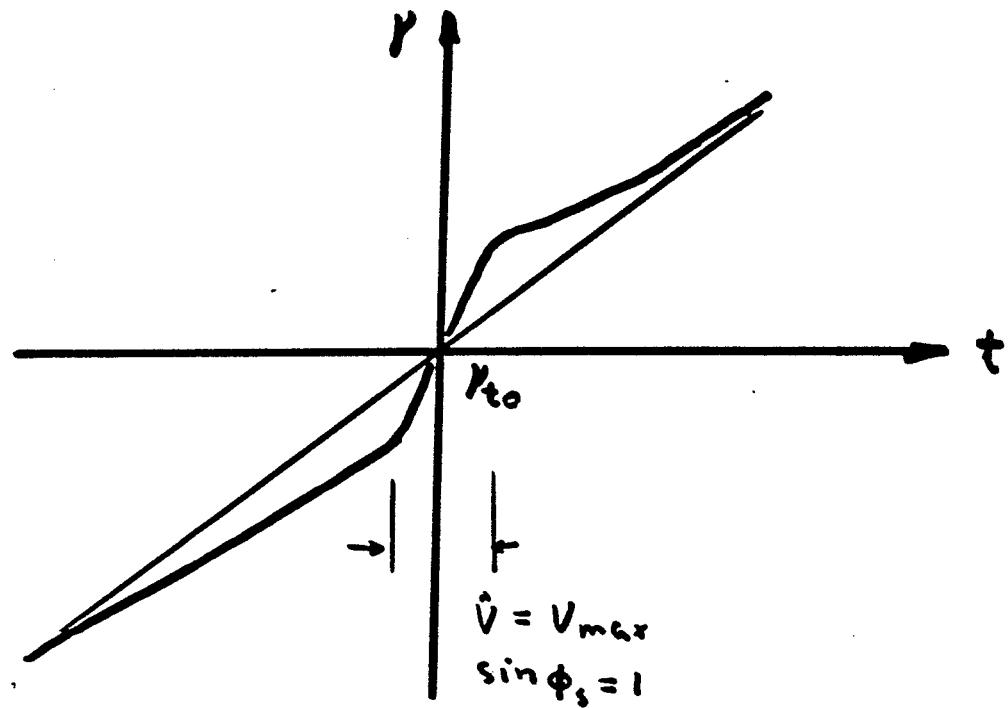
Efficiency 70%



$+290 \text{ ms}$

7200 particles used.

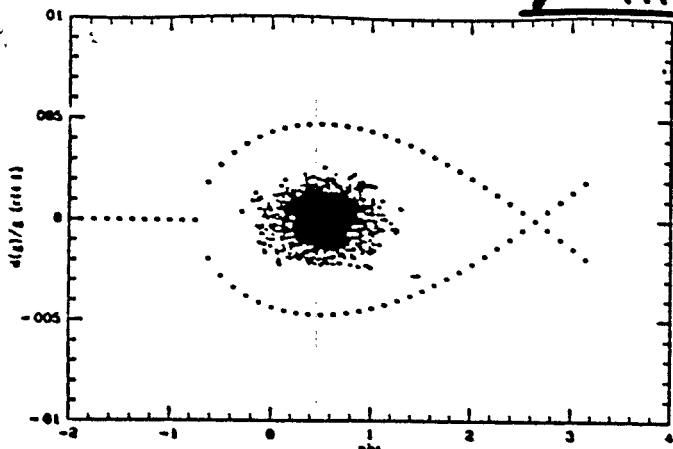
γ increase crossing transition



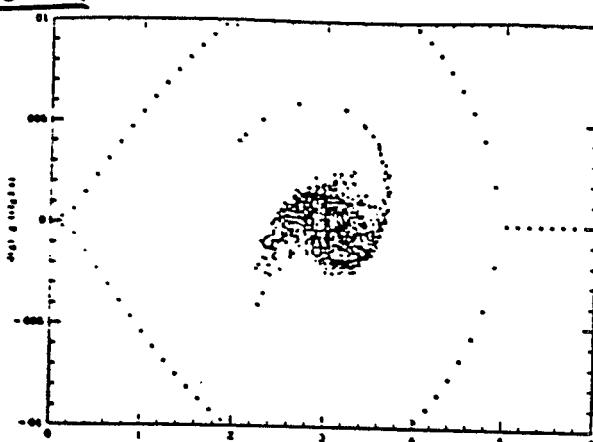
Total momentum aperture needed $\pm 0.8\%$

$\Delta\gamma = 0.38$ in 40 ms. used in simul.

INCREASE \downarrow \rightarrow 300KV

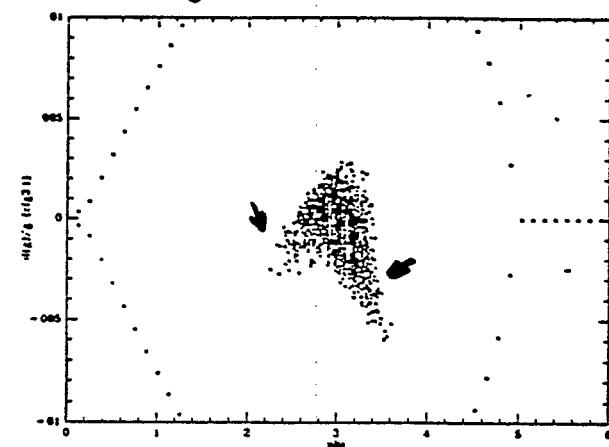


$\downarrow -150 \text{ ms}$

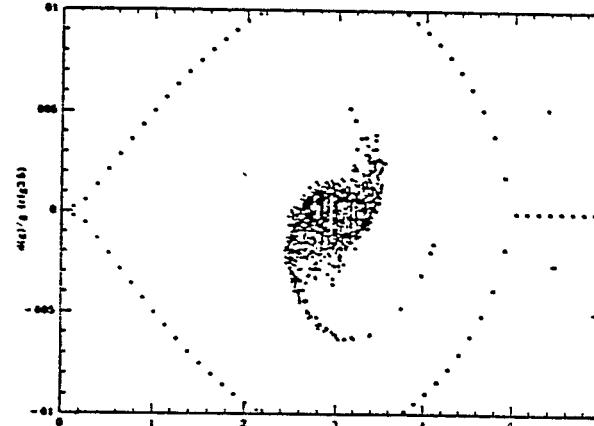


$\downarrow +175 \text{ ms}$

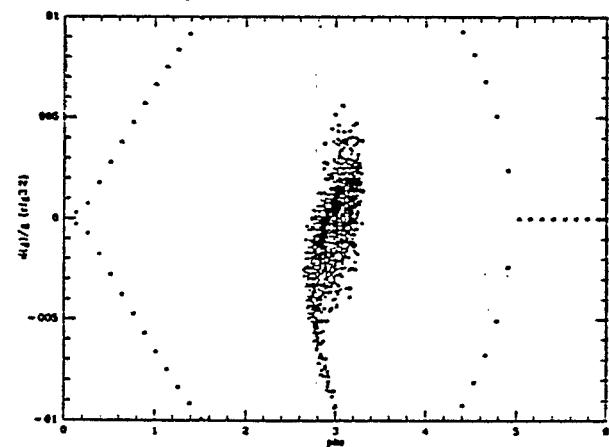
A_u^{+79}
0.3 eV·s/a
 γ increase
 $\alpha = -0.6$
 $\frac{z}{n} = 1.2 \Omega$
capac.



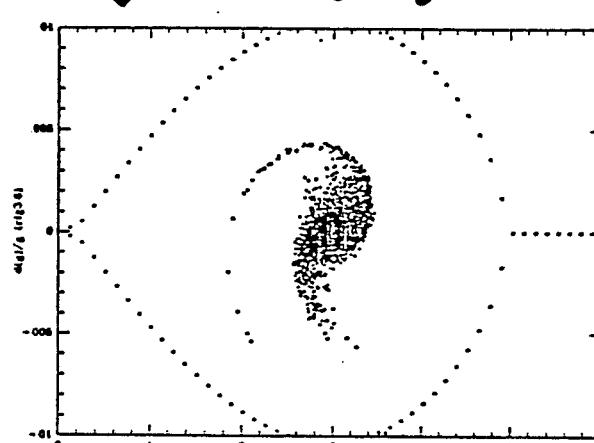
$\downarrow +25 \text{ ms}$



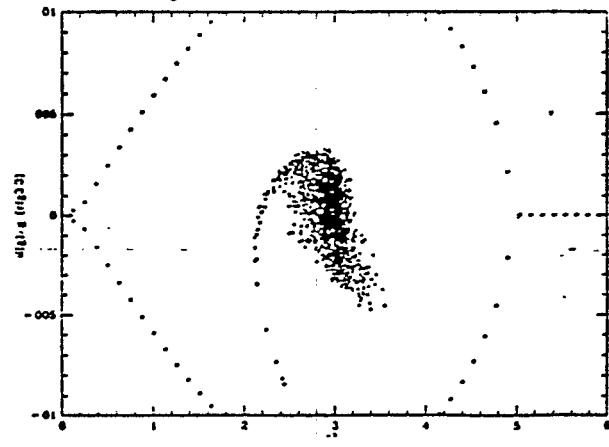
$\downarrow +225 \text{ ms}$



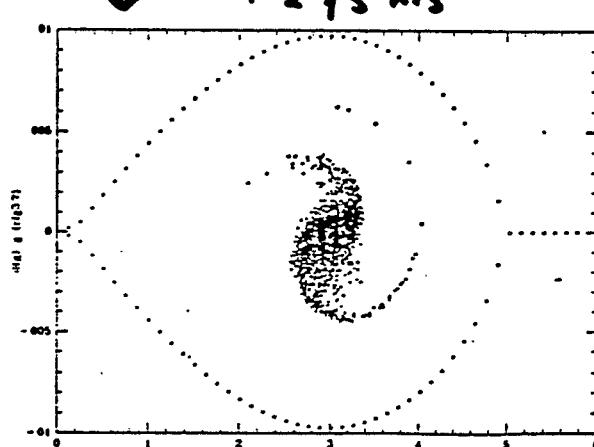
$\downarrow +75 \text{ ms}$



$\downarrow +275 \text{ ms}$



$\downarrow +125 \text{ ms}$



$\downarrow +325 \text{ ms}$

Eff. 100%
0.3 ~ 0.4
eV·s/a

$\hat{V} = 100 \text{ kV}$

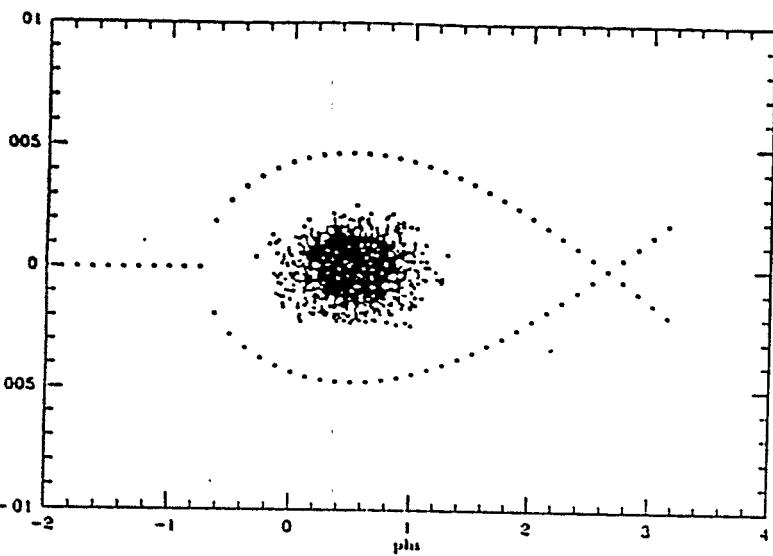
$\sin \phi_s = 0.48$

γ_t JUMP

$\Delta \gamma_t = 0.6$

in 60 ms

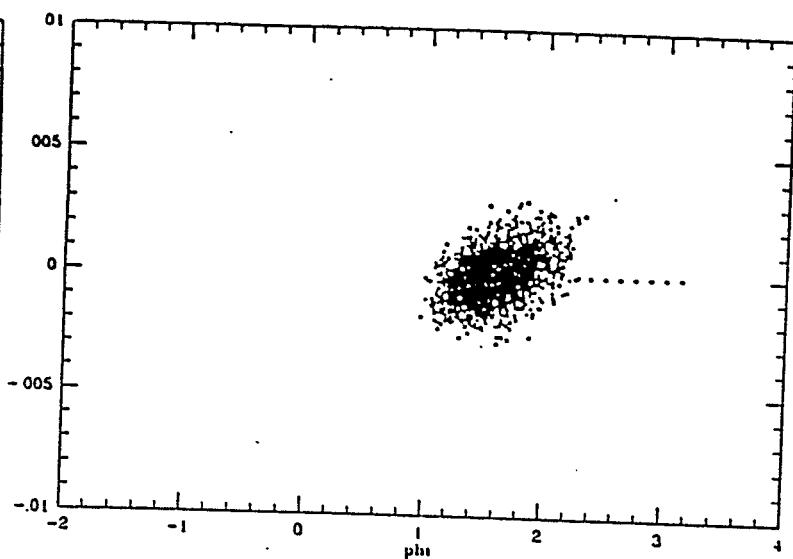
Au^{+79}



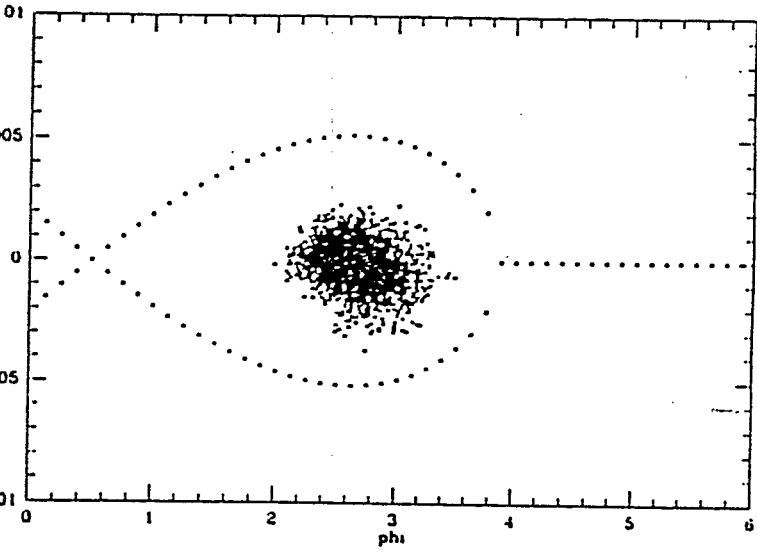
-80 ms

$0.3 \text{ eV} \cdot \text{s}/\text{amu}$

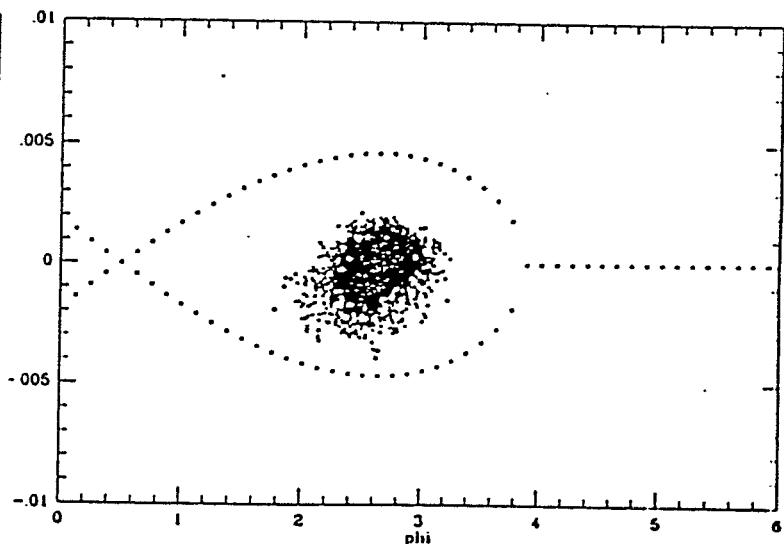
$$\frac{Z}{n} = 1.2 \text{ } \Omega$$



0 ms



$+25 \text{ ms}$



$+80 \text{ ms}$

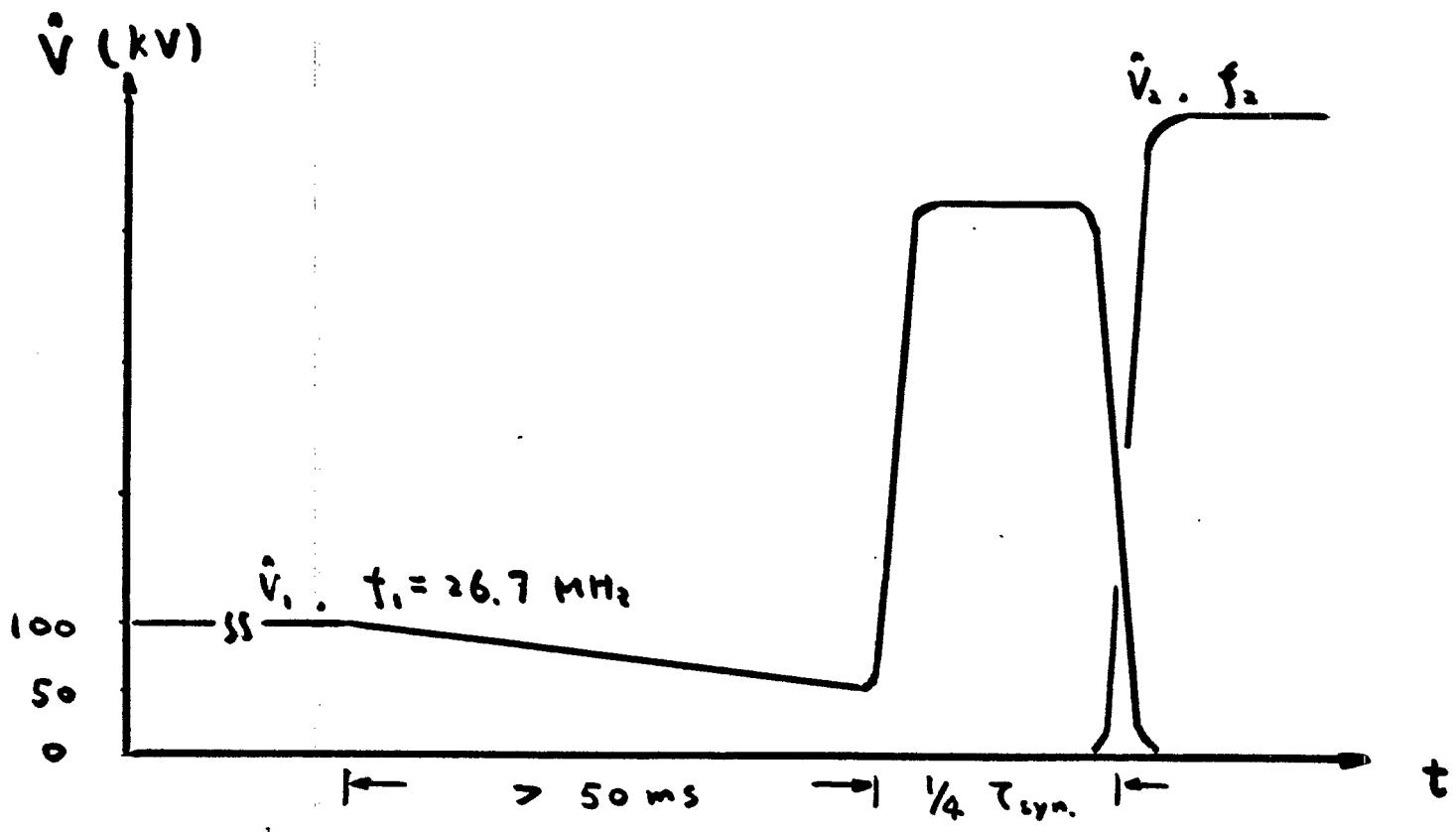
$\eta(\frac{\Delta \rho}{P}) \& \text{ s.c. included. No Loss}$

$\tau_0 \ll T_{n,i} \ll T_c < \tau_{syn.}$

Negligible Blow up

II. Transfer to High Freq. RF System

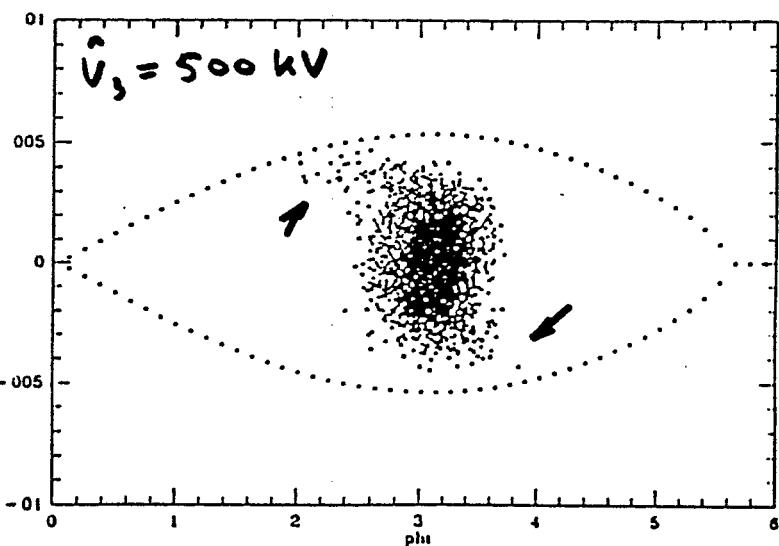
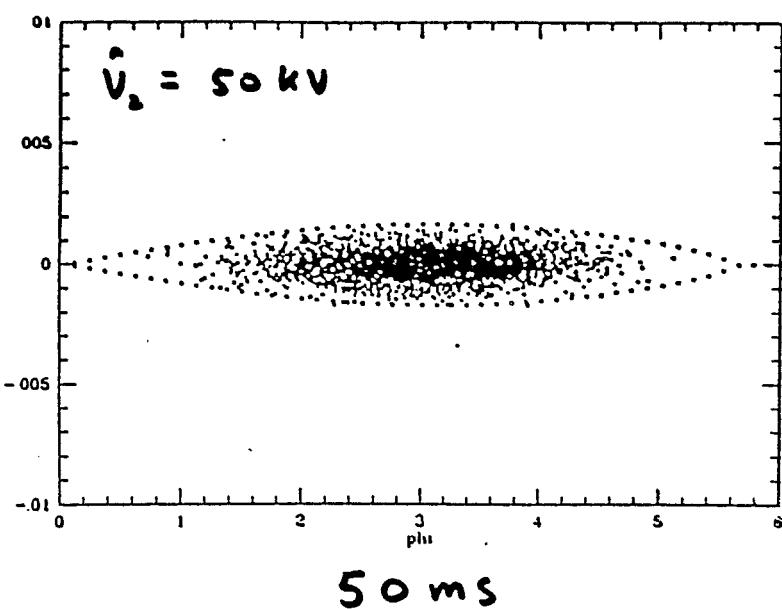
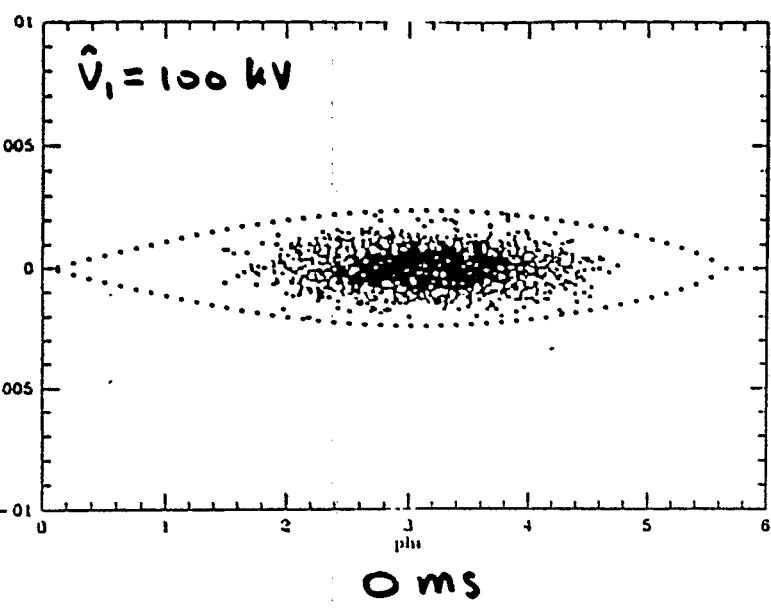
- $1 \text{ eV}\cdot\text{s}/\text{amu}$ bunch area. (general case)
Bunch rotation (squeeze $\frac{\partial p}{p}$, rotate, recapture)
- $0.3 \text{ eV}\cdot\text{s}/\text{amu}$ bunch area
 - * Adiabatic compression , A. G. Ruggiero
 - * "Simple" rotation , E. Raka
 - * Unstable fixed point rotation , S. Y. Lee
J. Wei
- Switch over near transition
proposed by J. M. Brennan
 - ✓ , when combined with γ_t jump
or γ increase
- 160 MHz, right bucket length



Bunch rotation

Bunch Rotation, 1 eV.s/a

A_u^{+79}

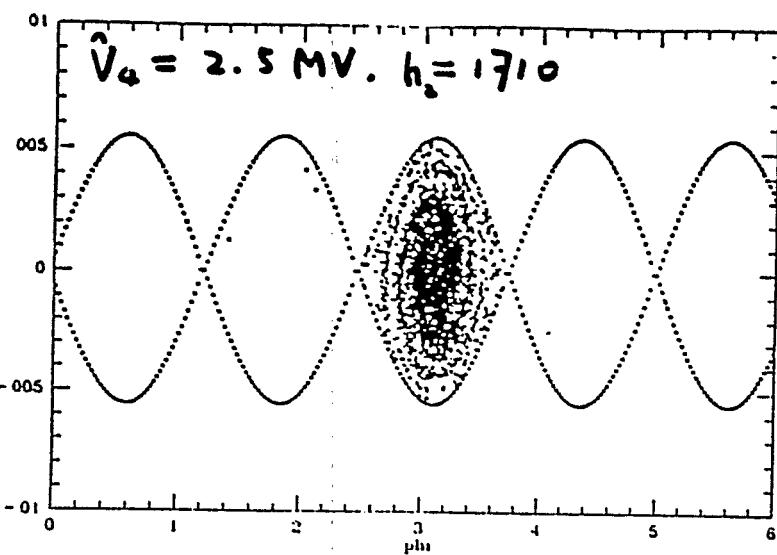


1 eV.s/gm_u
 $\gamma_{top} = 30$
with S.C.

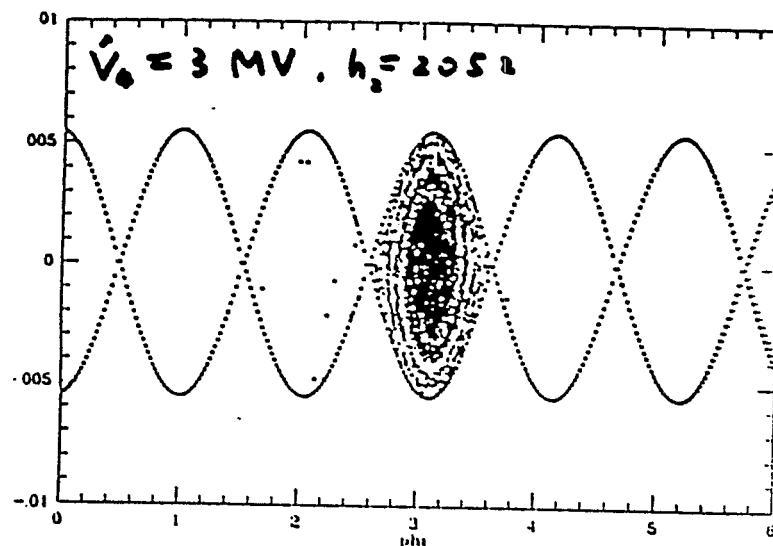
58 ms

Bunch Rotation Recapture

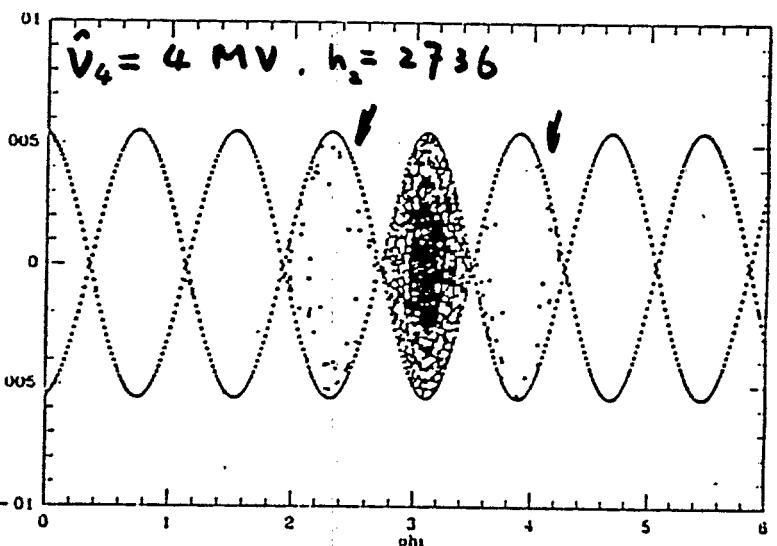
Au^{+79}
RHIC



134 MHz 98%
survival



160 MHz 95%
surv.



214 MHz 87%
surv.

$$\hat{V}_3 = 500 \text{ kV}$$

$$\hat{V}_4 = \frac{h_2}{h_1} \hat{V}_3$$

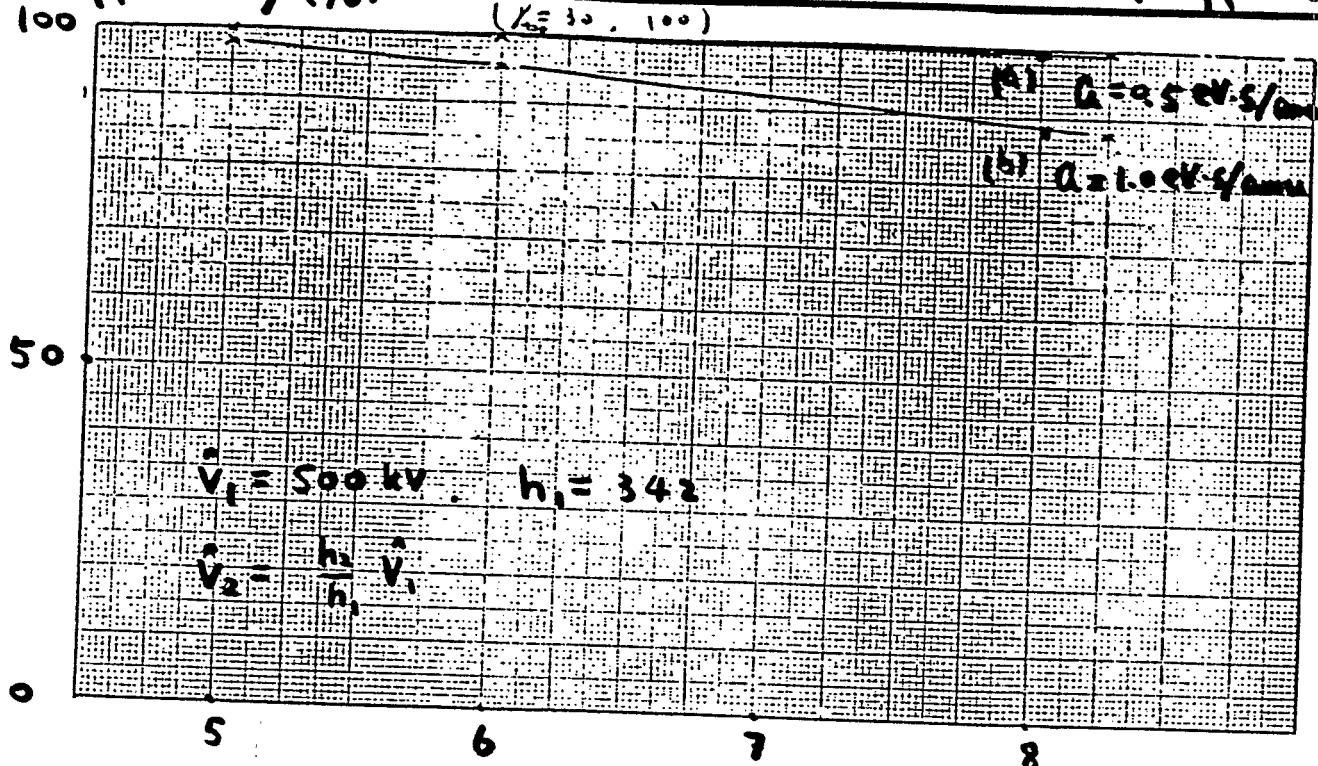
$$h_1 = 342$$

(At $\gamma_{t4} = 30$)

Efficiency (%)

Top Energy Bunch Rotation Efficiency

($\lambda = 30, 100$)

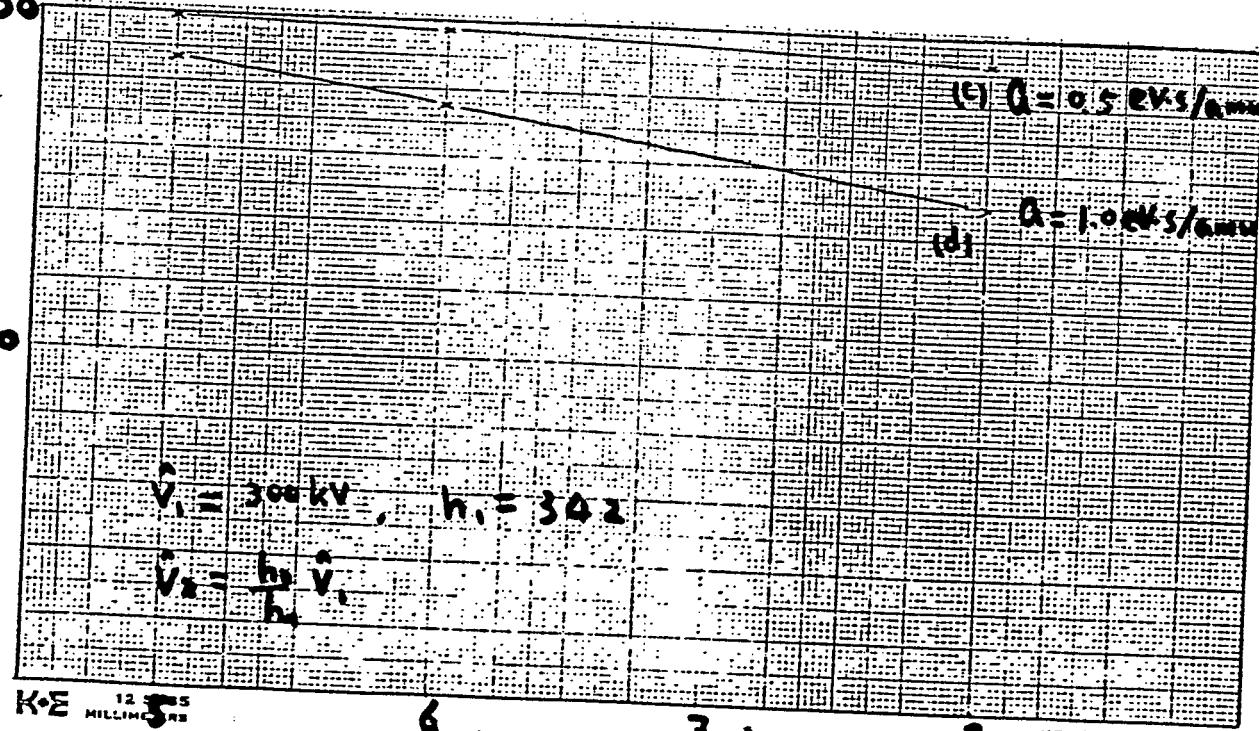


a) $\hat{V}_1: 100\text{kV} \rightarrow 15 \text{ kV} \rightarrow 500 \text{ kV}$

b) $\hat{V}_1: 100\text{kV} \rightarrow 50 \text{ kV} \rightarrow 500 \text{ kV}$

Efficiency (%)

100



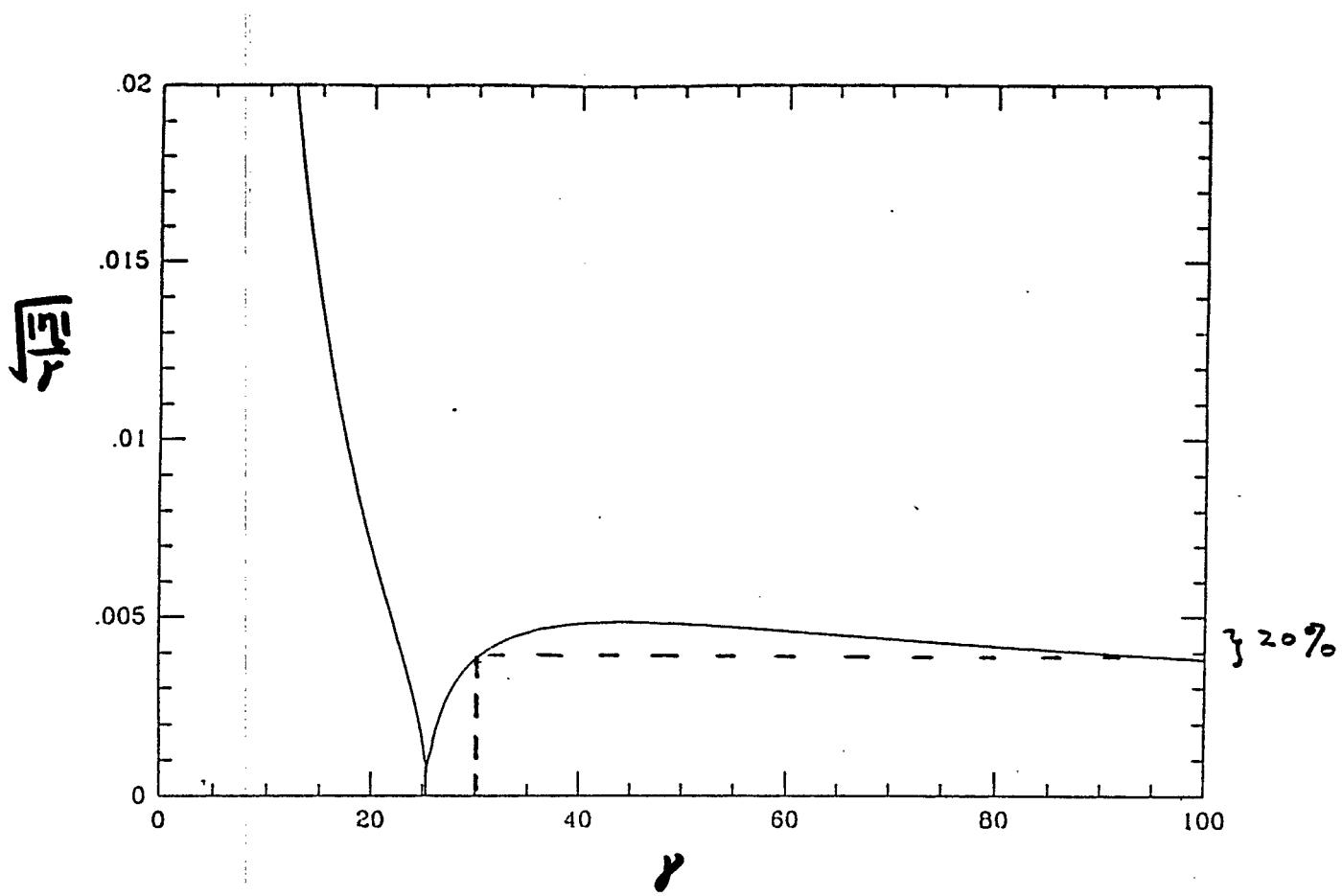
c) $\hat{V}_1: 100\text{kV} \rightarrow 15 \text{ kV} \rightarrow 300 \text{ kV}$

d) $\hat{V}_1: 100\text{kV} \rightarrow 50 \text{ kV} \rightarrow 300 \text{ kV}$

KEUFFEL & ESSER CO.
MADE IN U.S.A.

$\frac{h_2}{h_1}$

Energy dependence of Rotation

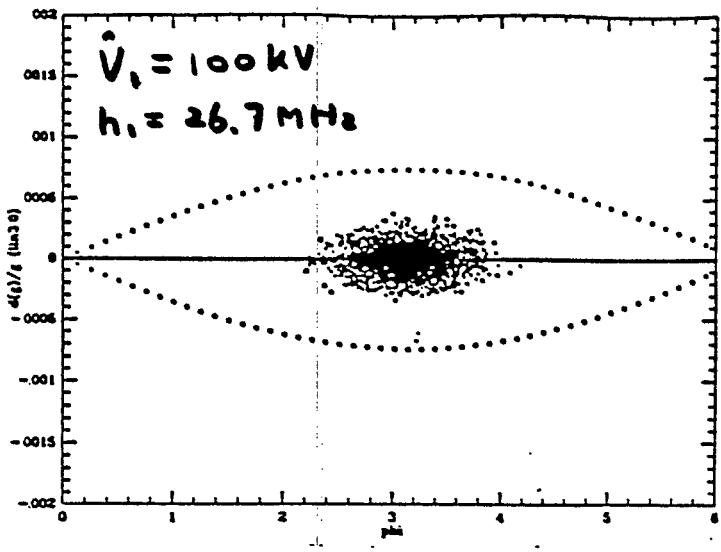


$$\Omega_s \propto \sqrt{\frac{\eta_1}{\eta}} \quad \text{timing}$$

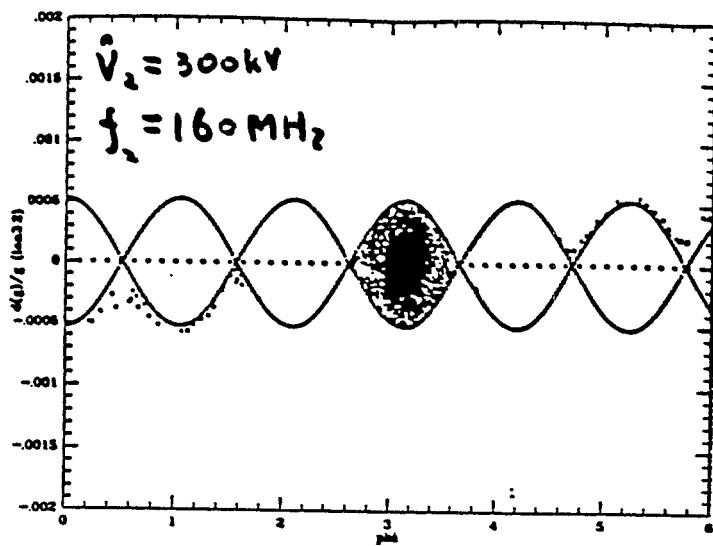
$$\hat{V}_s \propto \sqrt{\frac{\eta_1}{\eta}} \quad \text{rotation voltage}$$

0.3 eV·s/amu "Simple" rotation

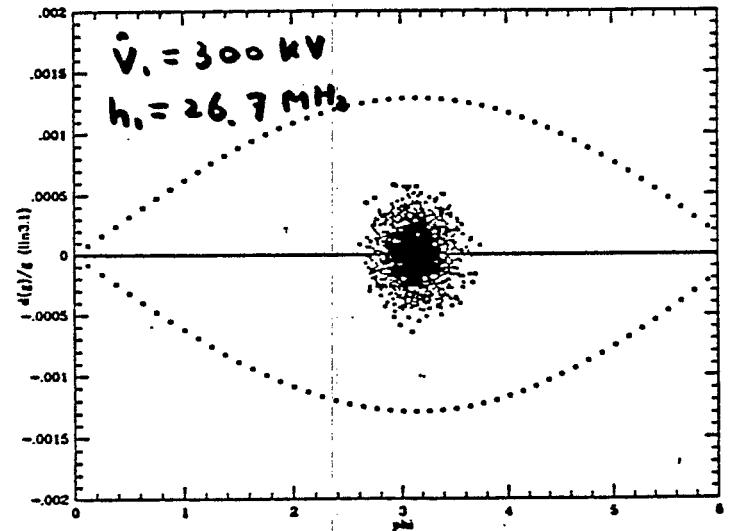
Au
RHIC



0 ms



36 ms



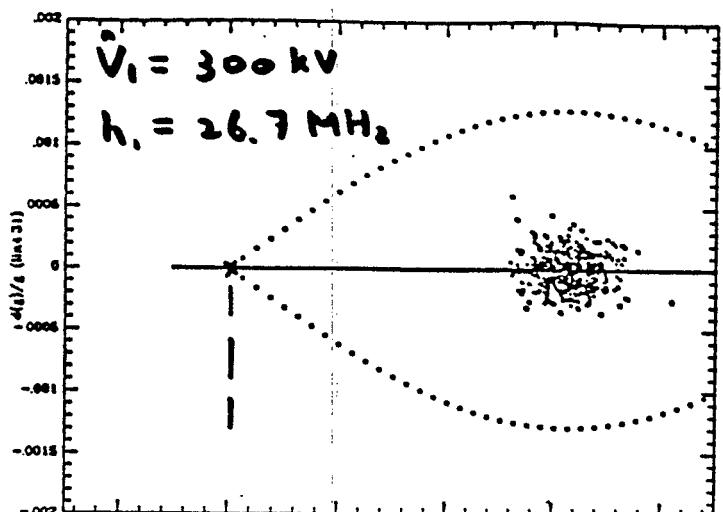
10 ms

98% in 0.39 eV·s/amu
bucket

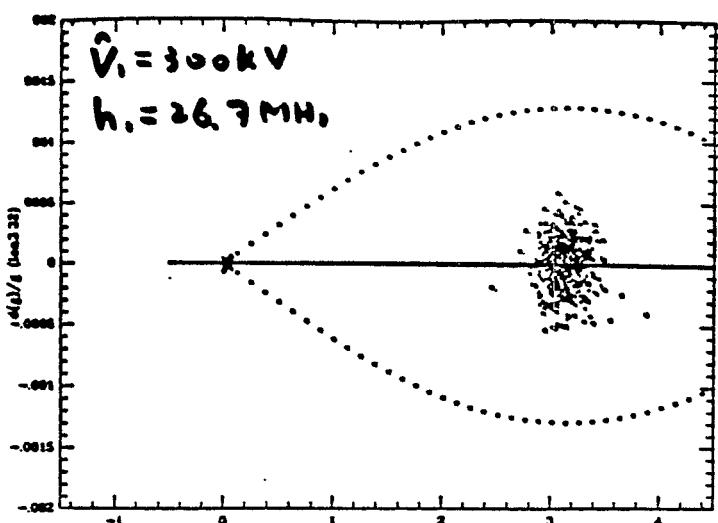
(At $\gamma_{\text{top}} = 100$)

Rotation by using unstable fixed point

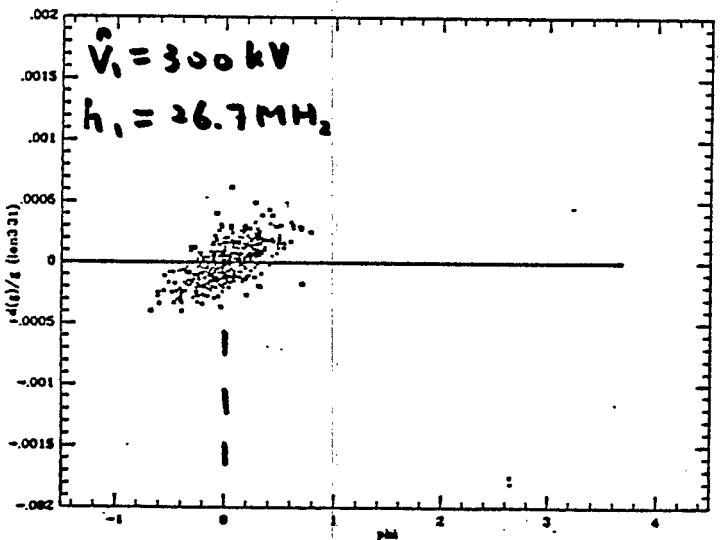
Au^{79}
RHIC



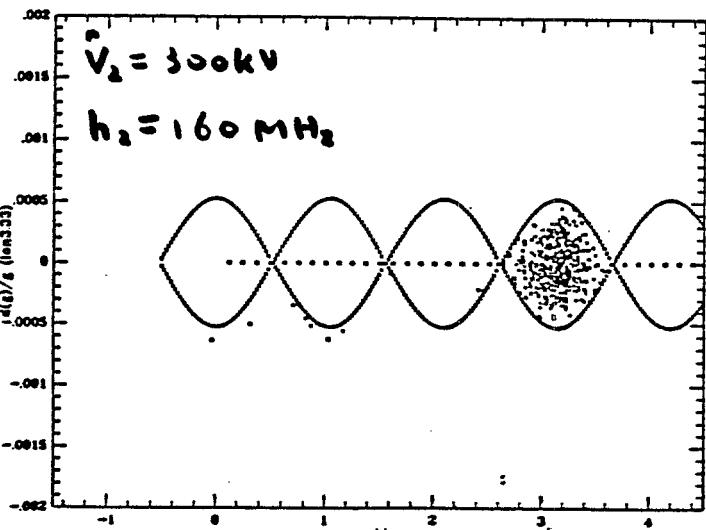
0 ms



14 ms



2.6 ms



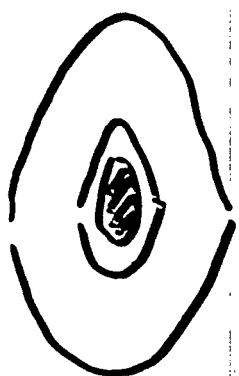
43 ms

98% in $0.39 \text{ eV}\cdot\text{s}/\text{amu}$

bucket

(At $\gamma_{top} = 100$)

When matched



$$\frac{\Delta P}{P} / \frac{\Delta \phi}{\phi} \sim \hat{V}^{1/2} \cdot h^{1/2} \cdot \eta^{-1/2}$$

"
const

$$\begin{aligned}\hat{V}_1 &= 300 \text{ kV.} & h_1 &= 342 \\ \hat{V}_2 &= 300 \text{ kV.} & h_2 &= 6 * 342\end{aligned}$$

a gain of $\frac{\hat{V}_2}{\hat{V}_1} = 6$

AGS

$$\hat{V}_1 = 12 \text{ kV} \Rightarrow \hat{V}'_1 = 100 \text{ kV.}$$

$$\frac{\hat{V}'_1}{\hat{V}_1} \sim 8$$

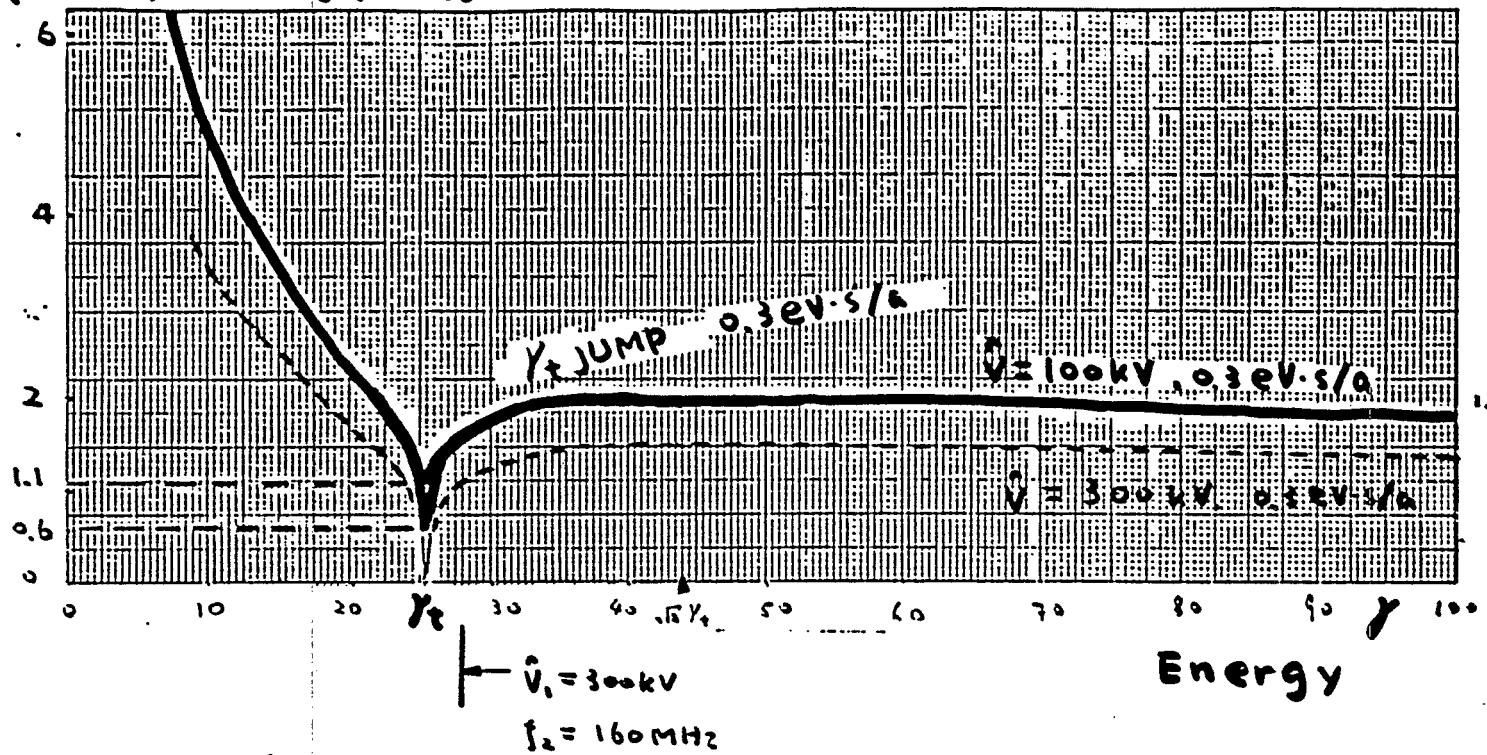
But

$$R_s = \sqrt{\frac{2eVh\eta c^3}{2\pi Am_0c^2 \cdot R_0} \cos q_s}$$

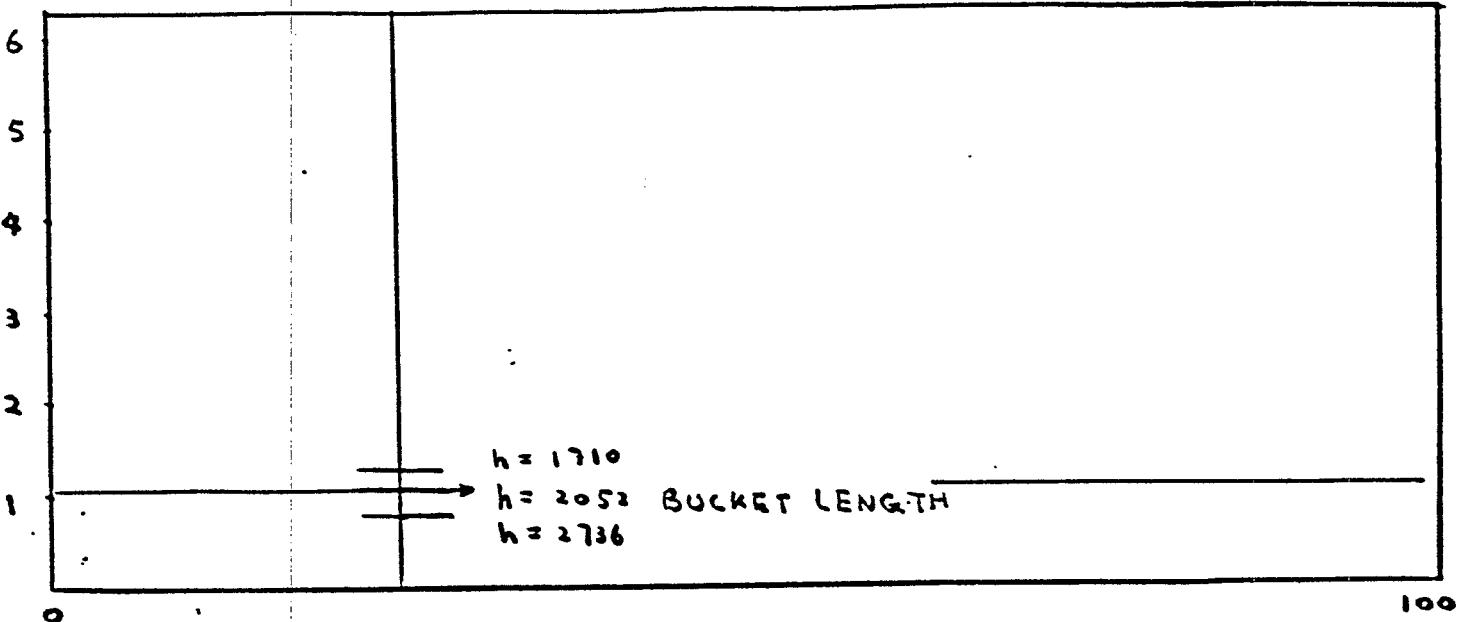
$$\Rightarrow \tau_{syn} = 5 \text{ ms} ?$$

RHIC, $\tau_{syn} = 40 \text{ ms. (300 kV)}$

Bunch Length (rad. 26.7 MHz phase)

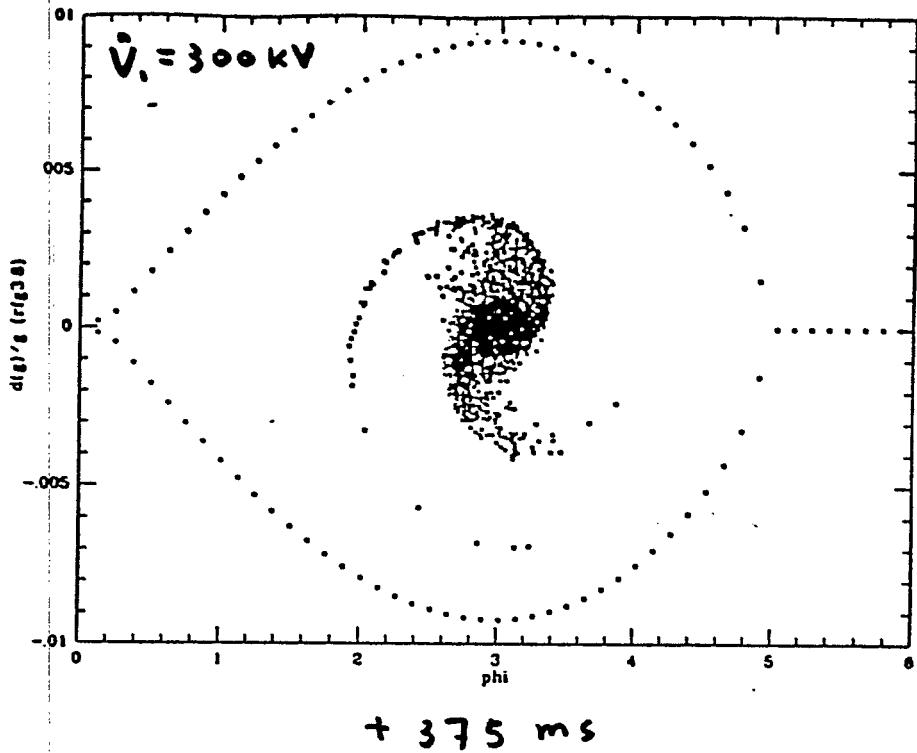


- Switch over near transition γ_t .

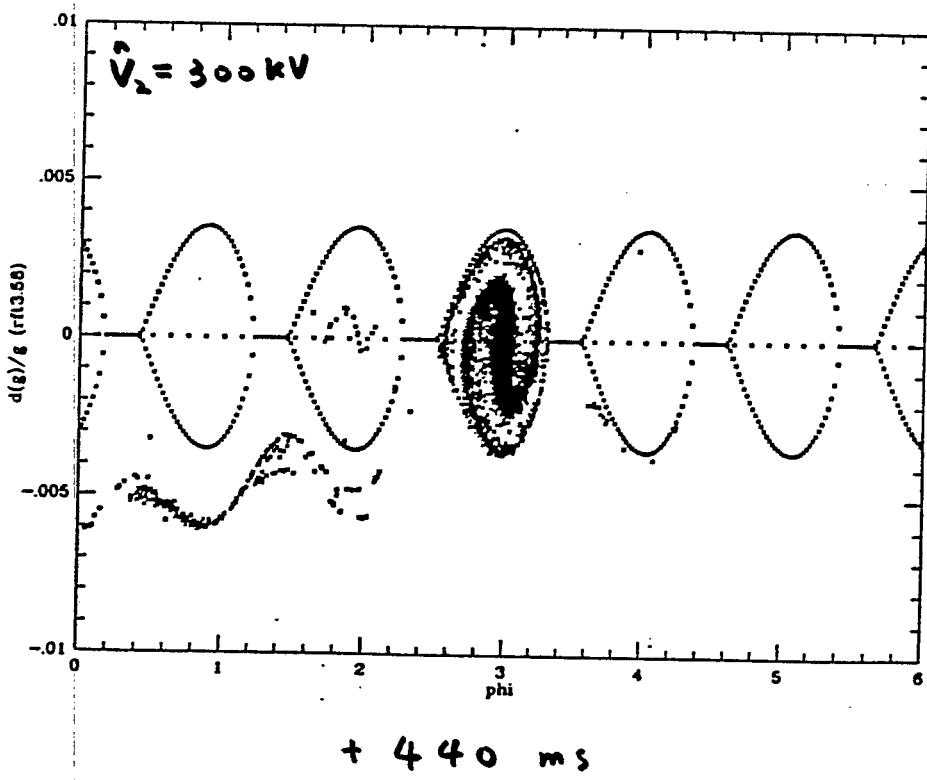


Switch over near transition

(after a γ increase)

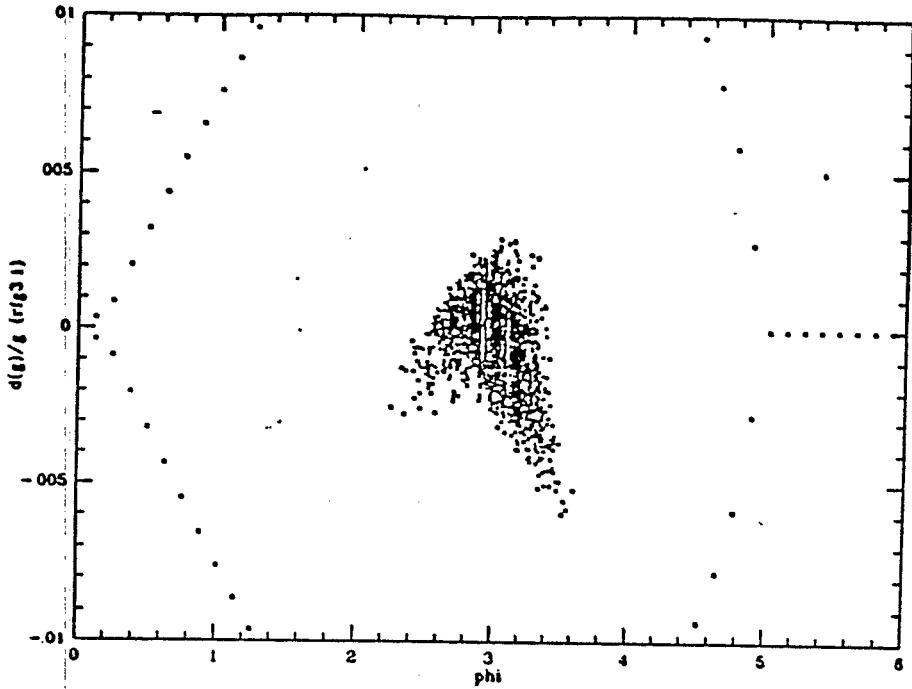


A_u^{+79}
 $\gamma_{tr.} = 26.14$
 $t_0 = 25.4376$
 $\hat{V}_1 = 100 \sim 300 \text{ kV}$
 $\hat{V}_2 = 300 \text{ kV}$
 $h_1 = 342$
 $h_2 = 2052$
 $\alpha_1 = -0.6$
 $\frac{2}{n} = 1.2 \pi$
 capac.

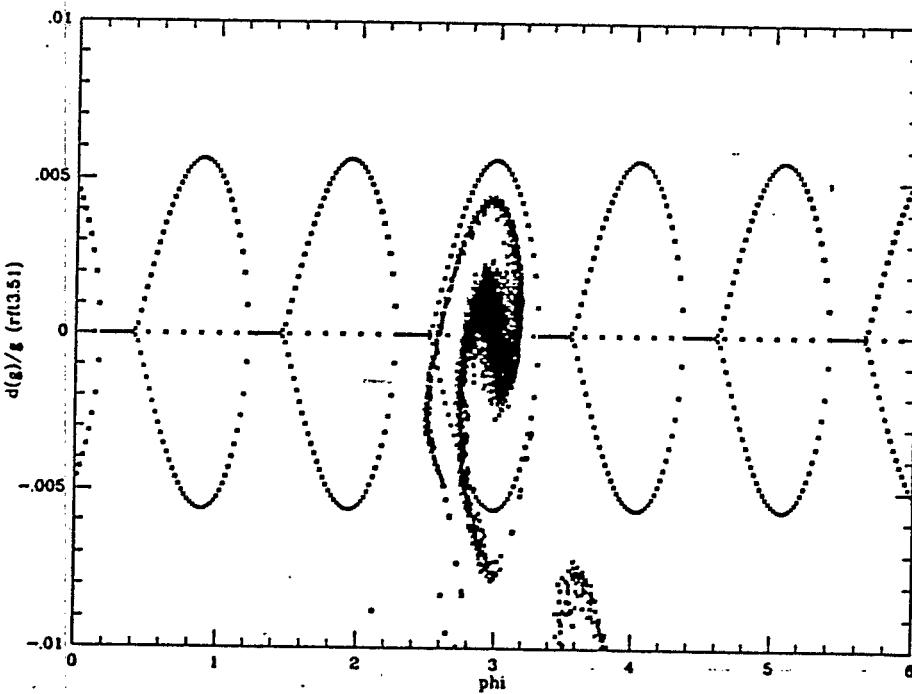


Transfer
Efficiency
90%
in $0.6 \text{ eV}\cdot\text{s}/\text{am}$

Switch over near transition
 (after a γ increase)



+ 25 ms



+ 90 ms

A^{79}
 $\gamma_{tr.} = 25.65$
 $\gamma_{t_0} = 25.4376$
 $\hat{V}_1 = 100 \sim 300 \text{ kV}$
 $\hat{V}_2 = 300 \text{ kV}$
 $h_1 = 342$
 $h_2 = 2052$
 $\alpha_1 = -0.6$
 $\frac{2}{n} = 1.2 \Omega$
 capac.

Transfer
Efficiency
83 %
in 0.6 eV·s/a

Transfer Efficiency

100%

* $\hat{V}_2 = 300 \text{ kV}$

* $\hat{V}_3 = 600 \text{ kV}$

* $\hat{V} = 2 \text{ MV}$

$\dot{\gamma}$ increase crossing γ_t

transfer to $\hbar = 20.5$ system

$$\alpha_1 = -0.6$$

$$\frac{2}{n} = 1.2 \text{ n capacitive}$$

50

X to 25.5

25.60

25.70

25.80

25.90

26.0

26.10

26.20

γ_{transfer}

* $\dot{\gamma}$ increase crossing γ_t

$\hat{V}_1 = 100 \text{ kV} \rightarrow 300 \text{ kV}$

$\hat{V}_2 = 2 \text{ MV}$

$\hat{V}_3 = 600 \text{ kV}$

$\hat{V}_3 = 300 \text{ kV}$

$\sin \Phi_1 = 0.024$, $A_2 = 2.3 \text{ evs/c}$

$\sin \Phi_1 = 0.08$, $A_2 = 1.2 \text{ evs/c}$

$\sin \Phi_1 = 0.16$, $A_2 = 0.6 \text{ evs/c}$

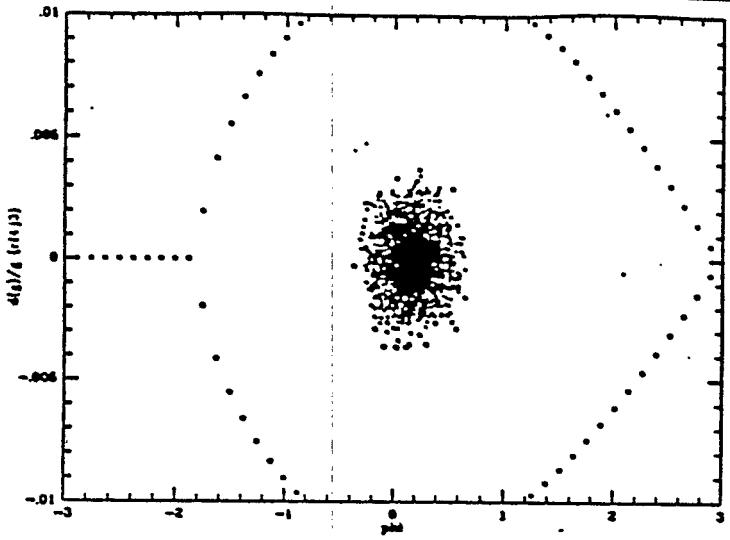
- $\dot{\gamma}$ increase of $\Delta \gamma = 0.38$. in 40 ms
- then transfer to 160 MHz RF System

On $\gamma = \gamma_{\text{transfer}}$

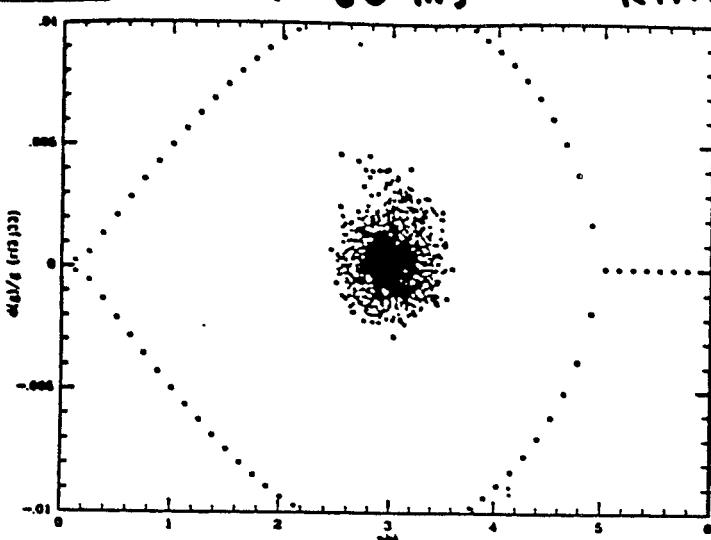
γ_t jump, $V_0 = 300$ kV

$\Delta\gamma_t = 0.6$
in 60 ms

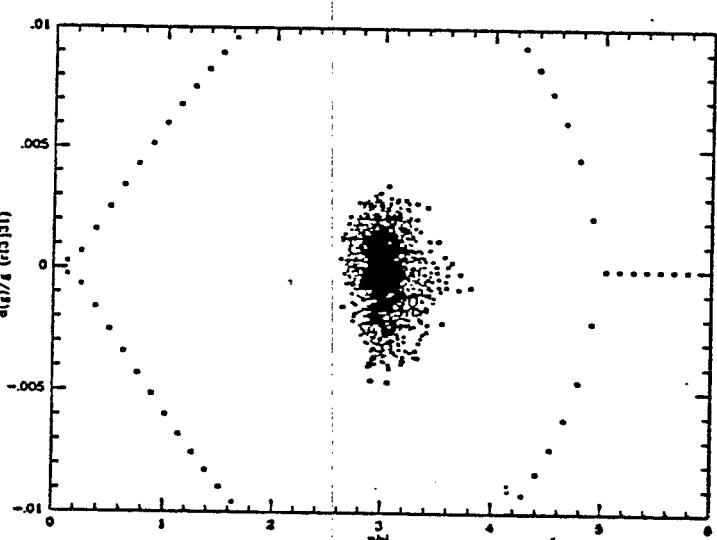
Au⁷⁷
RHIC



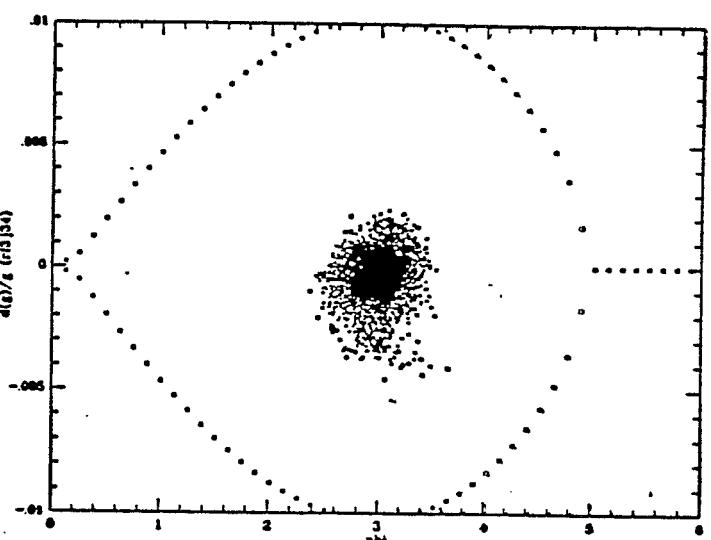
-80 ms



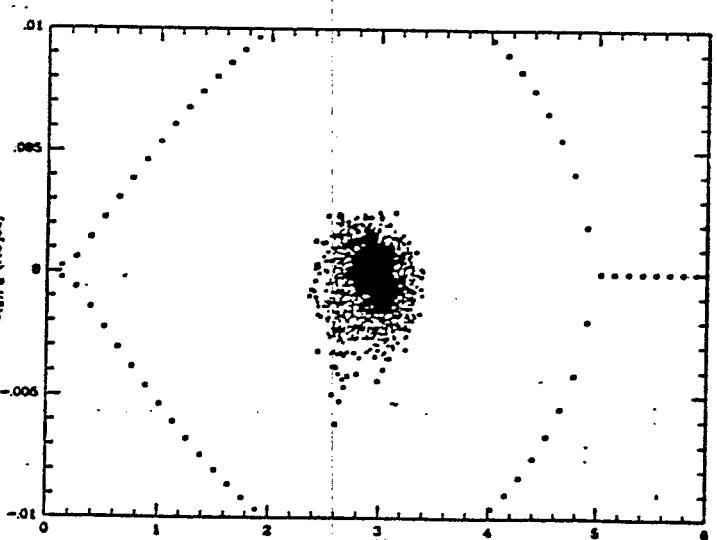
$+125$ ms



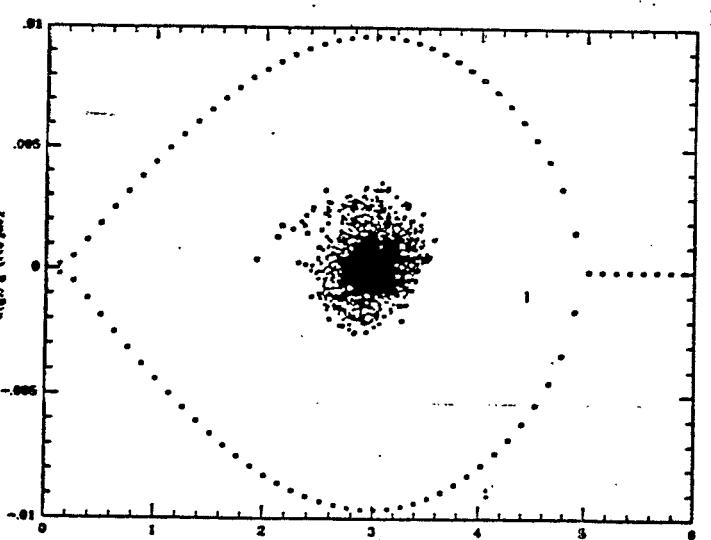
$+25$ ms



$+175$ ms



$+75$ ms

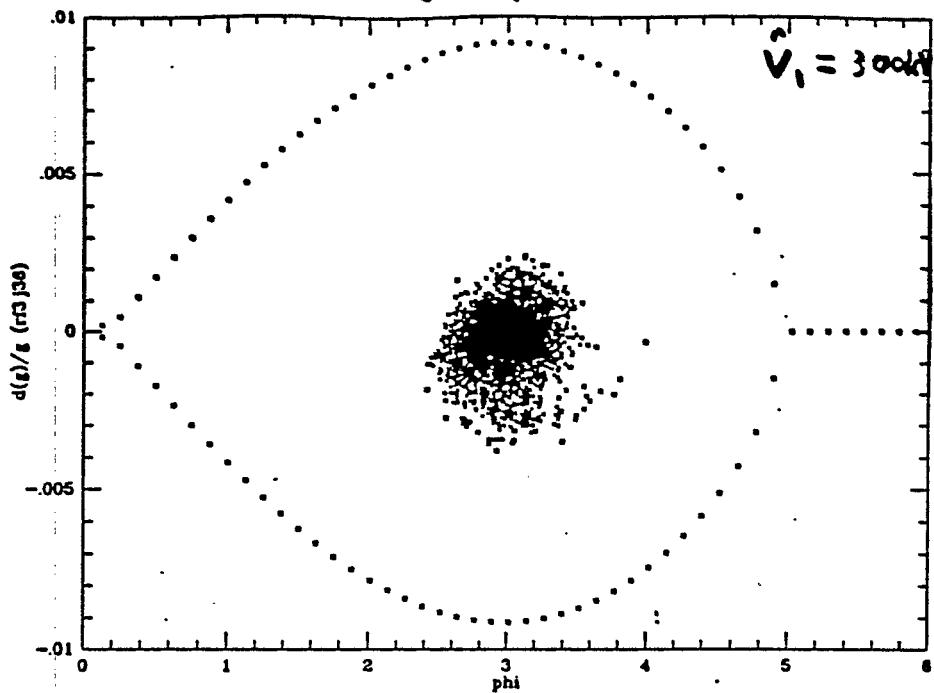


$+225$ ms No Loss

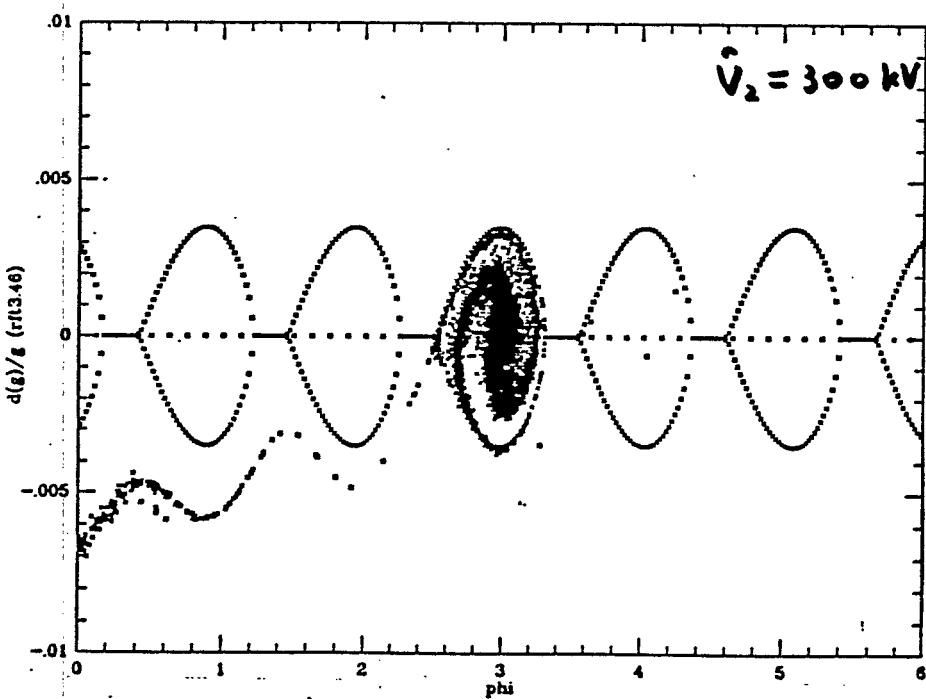
Switch over near transition

(after a γ_t jump)

Au⁺⁷⁹
RHIC



+ 275 ms



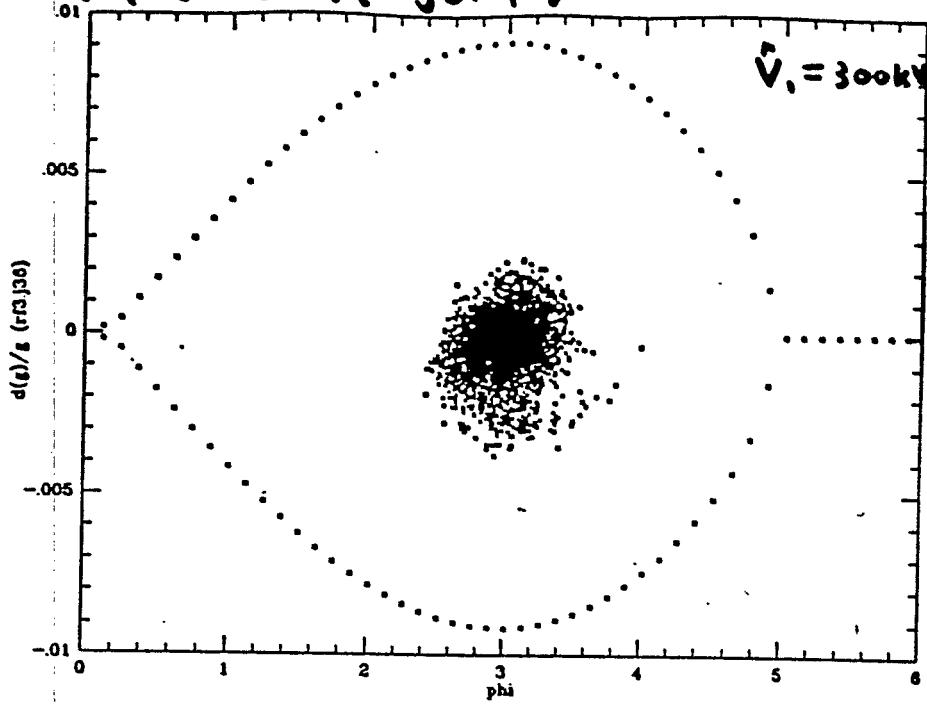
+ 340 ms

92% surv.
in 0.6 ev. s/c
bucket

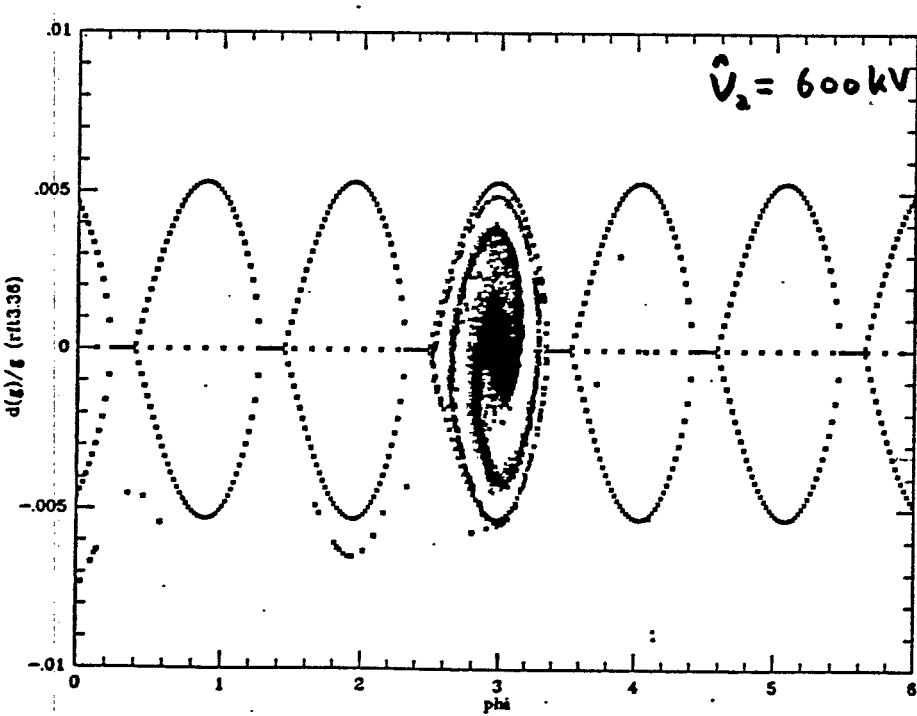
Switch over near transition

(after a γ_t jump)

Au⁺⁷⁹
RHIC



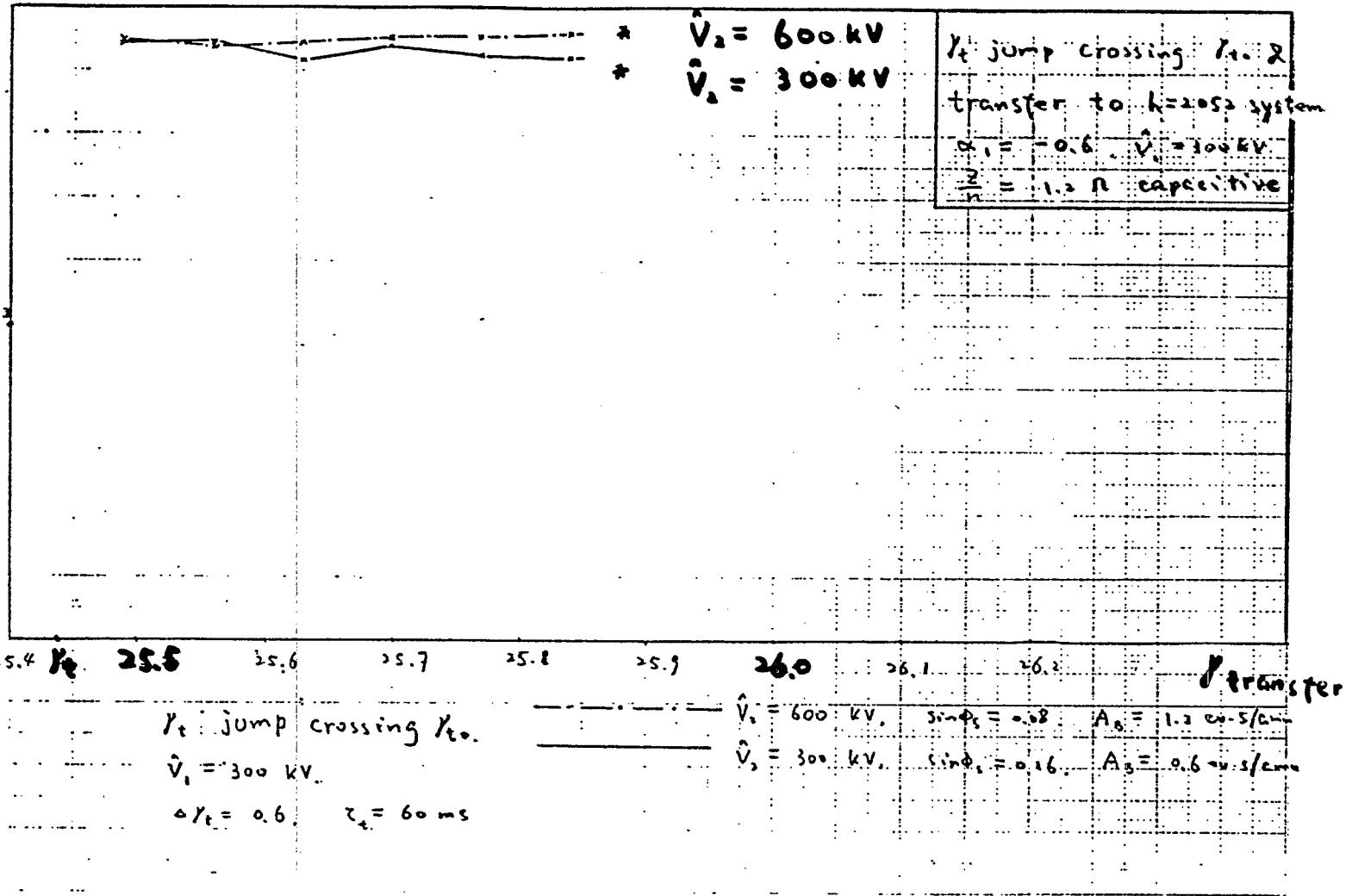
+ 275 ms



+ 340 ms

96% surv.
in 1.2 eV.s/a
bucket

Transfer efficiency



- Y_t jump of $\Delta Y_t = 0.6$ in 60 ms , $\hat{V}_1 = 300 \text{ kV}$
- then transfer to 160 MHz RF System
on $\gamma = \gamma_{\text{transfer}}$

Summary on the previous study:

- * A γ_t jump, or a $\dot{\gamma}$ increase near transition is very helpful
- * A r.f. voltage of 300 kV for 26.7 MHz system helps in achieving $\dot{\gamma}$ increase, rotation and providing sufficient bucket area at low energy
- * A second system of 160 MHz is comfortable for r.f. system transfer.